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TIME DOMAIN FUNDAMENTAL SOLUTION TO BIOT'S COMPLETE EQUATIONS OF DYNAMIC POROELASTICITY PART II: THREE-DIMENSIONAL SOLUTION

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Abstract—To complete the fundamental solutions for Biot's theory of dynamic poroelasticity, this paper is dedicated to the three-dimensional case. The solutions in the Laplace transform domain are presented and the Green's functions for elastodynamics and steady-state poroelasticity are easily recovered as the special cases of the present solutions. Both the transient solutions for the limiting case and for the general case have been derived. Lastly, the variations with time of the solid displacements and fluid pressure components for the point loads after the arrival of the waves are studied graphically, the ensuing transient Green's function components are compared with Laplace transform domain solutions and are found in excellent agreement, except for the limiting case at later times as have been expected.

INTRODUCTION

In a companion paper (Chen, 1993a), the author has presented a complete set of fundamental solutions for two-dimensional Biot's (Biot, 1956a,b, 1962) full dynamic poroelasticity both in Laplace domain and in time domain. The solutions in time domain have been successfully constructed both for limiting case (short time approximation) and for general full dynamic case.

The objective of the present study is to complete the fundamental solutions pertaining to Biot's theory by providing its counterpart for the three-dimensional case. In addition to their mathematical attributes and the physical significance in revealing insights into wave propagations in poroelastic media, the time domain three-dimensional fundamental solution provides a key ingredient, combined with time domain boundary integral representation (Chen, 1993a), for development of three-dimensional time domain BEM. Such a solution will certainly hold an upper hand over its corresponding Laplace domain BEM (Chen, 1992; Chen and Dargush, 1993b,c) in solving more complicated problems of dynamic nonlinear poroelasticity, dynamic soil-structure interaction, seismic wave scattering, earthquake engineering, acoustics and biomechanics.

REVIEW OF GOVERNING EQUATIONS

The three-dimensional governing equations for dynamic poroelasticity (Biot, 1956a,b; Zienkiewicz *et al.*, 1980; Zienkiewicz and Shiomi, 1984) are repeated below for reference.

The first constitutive relation:

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) - \alpha p\delta_{ij}. \tag{1}$$

The second constitutive relation:

$$\theta = \alpha u_{k,k} + \frac{1}{Q}p. \tag{2}$$

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The equilibrium equation:

$$\sigma_{ij,j}+f_i = \rho\ddot{u}_i+\rho_f\ddot{w}_i. \tag{3}$$

The generalized Darcy's law:

$$\dot{w}_i = -\kappa(p_{,i}+\rho_f\ddot{u}_i+m\ddot{w}_i). \tag{4}$$

The continuity equation:

$$\dot{w}_{k,k} = -\alpha\dot{u}_{k,k}-\frac{1}{Q}\dot{p}+\gamma, \tag{5}$$

where $i,j = 1,2,3$, σ_{ij} is the total stress, p denotes the excessive fluid pore pressure (pressure is taken as positive), u_i is the displacement of the solid skeleton, w_i denotes the average displacements of the fluid relative to the solid, $\dot{w}_i = (\mathrm{d}w_i/\mathrm{d}t)$ is the average relative velocity of seepage measured over the total area, θ represent the increment of fluid content. The elastic constants λ and μ are termed drained Lamé constants, $\kappa = (k/\eta)$ is the permeability coefficient, while η and k denote the fluid dynamic viscosity and the intrinsic permeability of the solid skeleton, respectively, ρ_f is fluid density and $m = \rho_f/n$ (Zienkiewicz *et al.*, 1980) or $m = (\rho_a/n^2)+(\rho_f/n)$ (Biot, 1956a; Bonnet and Auriault, 1985; Cheng and Badmus, 1991), where n is the porosity, ρ_a is the apparent mass density corresponding to the work done by the solid phase to the fluid phase due to the relative motion between them. In addition α and Q are Biot's parameters accounting for compressibility in the two-phase material, $\rho = (1-n)\rho_s+n\rho_f$ is the density of the solid-fluid mixture, while ρ_s, f_i and γ are the density of the solid material, the body force and the rate of fluid injection, respectively. The governing equations in the form of the field equation and the energy equation, which involves four independent variables u_i and p, can only be expressed in the transform domain. That is:

$$(\lambda+\mu)\tilde{u}_{j,ij}+\mu\tilde{u}_{i,jj}-\alpha_1\tilde{p}_{,i}-\rho_1 s^2\tilde{u}_i+\tilde{f}_i = 0, \tag{6}$$

$$\zeta\tilde{p}_{,ii}-\frac{s}{Q}\tilde{p}-\alpha_2 s\tilde{u}_{i,i}+\tilde{\gamma} = 0, \tag{7}$$

where $i,j = 1,2,3$, the tilde denotes the Laplace transformation, $\alpha_1 = \alpha-\rho_f s\zeta$, $\alpha_2 = \alpha-\rho_f s\zeta$, $\rho_1 = \rho-\rho_f^2 s\zeta$, $\zeta = ((1/\kappa)+ms)^{-1}$ and s is the Laplace transform parameter.

Equations (6) and (7) are nondimensionalized by using the parameters:

$$\xi_i = \frac{x_i}{\rho\kappa C_P} \quad \text{and} \quad \tau = \frac{t}{\rho\kappa}, \tag{8}$$

where C_P is the propagation speed associated with waves moving through the porous media without relative motion between the fluid and the solid phase given as:

$$C_P = \sqrt{\frac{\lambda+2\mu+\alpha^2 Q}{\rho}}. \tag{9}$$

Next, we define a dimensionless displacement and pore pressure through:

$$U_i = \frac{u_i}{\rho\kappa C_P}, \qquad P = \frac{p}{\rho C_P^2}, \tag{10}$$

and denote

$$\lambda^* = \frac{\lambda}{\lambda+2\mu+\alpha^2 Q}, \tag{11a}$$

$$\mu^* = \frac{\mu}{\lambda+2\mu+\alpha^2 Q}, \tag{11b}$$

$$Q^* = \frac{Q}{\lambda+2\mu+\alpha^2 Q}, \tag{11c}$$

$$\rho^* = 1, \tag{11d}$$

$$\rho_f^* = \frac{\rho_f}{\rho}, \tag{11e}$$

$$m^* = \frac{m}{\rho}, \tag{11f}$$

$$\kappa^* = 1. \tag{11g}$$

The nondimensional form for the field equation (6) and the energy equation (7) is then:

$$(\lambda^*+\mu^*)\tilde{U}_{j,ij}+\mu^*\tilde{U}_{i,jj}-\alpha_1^*\tilde{P}_{,i}-\rho_1^* s^2\tilde{U}_i+\tilde{F}_i = 0, \tag{12}$$

$$\zeta^*\tilde{P}_{,ii}-\frac{s}{Q^*}\tilde{P}-\alpha_2^* s\tilde{U}_{i,i}+\tilde{\Gamma} = 0, \tag{13}$$

where $i,j = 1,2,3$, $\tilde{F}_i$ and $\tilde{\Gamma}$ are nondimensionalized body force and fluid source injection, α_1^*, α_2^*, ρ_1^*, ζ^* are defined by:

$$\alpha_1^* = \alpha_2^* = \alpha-\rho_f^* s\zeta^*,$$
$$\rho_1^* = \rho^*-(\rho_f^*)^2 s\zeta^*,$$
$$\zeta^* = \frac{1}{\dfrac{1}{\kappa^*}+m^* s}. \tag{14}$$

LAPLACE TRANSFORM DOMAIN FUNDAMENTAL SOLUTION

Once again following the same basic steps as shown in the two-dimensional case (Chen, 1993a), an explicit and well-posed three-dimensional Laplace transform domain fundamental solution for eqns (6) and (7), which involves the response to suddenly-applied three point forces and a supplementary scalar source with Heaviside step functions in time, can be obtained as follows:

$$\tilde{G}_{ij} = -(A_{ij}+B_{ij}\lambda_1+C_{ij}\lambda_1^2)\frac{\Lambda^2-\lambda_2^2}{\lambda_2^2-\lambda_1^2}\frac{1}{\rho_1 s^3}e^{-\lambda_1 r}$$
$$+(A_{ij}+B_{ij}\lambda_2+C_{ij}\lambda_2^2)\frac{\Lambda^2-\lambda_1^2}{\lambda_2^2-\lambda_1^2}\frac{1}{\rho_1 s^3}e^{-\lambda_2 r}$$
$$-(A_{ij}+B_{ij}\lambda_3+C_{ij}\lambda_3^2)\frac{1}{\rho_1 s^3}e^{-\lambda_3 r}+D_{ij}\frac{1}{s}e^{-\lambda_3 r}, \tag{15a}$$

$$\tilde{G}_{4i} = \frac{1}{4\pi(\lambda+2\mu)}\frac{\alpha_2}{\zeta}\frac{1}{\lambda_1^2-\lambda_2^2}\left(-\frac{x_i}{r^3}-\frac{x_i}{r^2}\lambda_1\right)e^{-\lambda_1 r}$$
$$+\frac{1}{4\pi(\lambda+2\mu)}\frac{\alpha_2}{\zeta}\frac{1}{\lambda_1^2-\lambda_2^2}\left(\frac{x_i}{r^3}+\frac{x_i}{r^2}\lambda_2\right)e^{-\lambda_2 r}, \tag{15b}$$

$$\tilde{G}_{i4} = \tilde{G}_{4i}\frac{1}{s}, \tag{15c}$$

$$\tilde{G}_{44} = \frac{1}{4\pi r\zeta s}\frac{\lambda_1^2-\Lambda^2}{\lambda_1^2-\lambda_2^2}e^{-\lambda_1 r} - \frac{1}{4\pi r\zeta s}\frac{\lambda_2^2-\Lambda^2}{\lambda_1^2-\lambda_2^2}e^{-\lambda_2 r}, \tag{15d}$$

where

$$\lambda_1^2+\lambda_2^2 = \Lambda^2 + \frac{s}{Q\zeta} + \frac{\alpha_1\alpha_2 s}{\zeta(\lambda+2\mu)}, \tag{16a}$$

$$\lambda_1^2\lambda_2^2 = \Lambda^2\frac{s}{Q\zeta}, \tag{16b}$$

$$\lambda_3^2 = \frac{\rho_1 s^2}{\mu}, \tag{16c}$$

$$\Lambda^2 = \frac{\rho_1 s^2}{\lambda+2\mu}, \tag{16d}$$

$$\zeta = \left(\frac{1}{\kappa}+ms\right)^{-1}, \tag{16e}$$

$$\alpha_1 = \alpha_2 = \alpha - \rho_f s\zeta, \tag{16f}$$

$$\rho_1 = \rho - \rho_f^2 s\zeta, \tag{16g}$$

and

$$i, j = 1,2,3, \quad r^2 = x_i x_i,$$

$$A_{ij} = \frac{1}{4\pi}\left(\frac{3x_i x_j}{r^5} - \frac{\delta_{ij}}{r^3}\right),$$

$$B_{ij} = \frac{1}{4\pi}\left(\frac{3x_i x_j}{r^4} - \frac{\delta_{ij}}{r^2}\right),$$

$$C_{ij} = \frac{1}{4\pi}\left(\frac{x_i x_j}{r^3}\right),$$

$$D_{ij} = \frac{1}{4\pi r\mu}\delta_{ij}. \tag{17}$$

Since the fundamental solution must obey radiation conditions, the roots λ_i $(i = 1,2,3)$ in eqns (15), whose real part is negative, must be rejected. $\tilde{G}_{ij}$ is the displacement of the solid skeleton in the i-th direction due to the unit Heaviside point force in the j-direction, whereas $\tilde{G}_{4j}$ is the fluid pressure due to the unit Heaviside point force in the j-th direction. Also $\tilde{G}_{i4}$ is the displacement of the solid skeleton in the i-th direction due to the unit Heaviside rate of fluid injection in fluid. And $\tilde{G}_{44}$ is the fuid pressure due to fluid injection.

It is interesting to see that in eqn (15a), i.e. the displacement generated by a point force, there are three waves present, namely a diffusive wave associated with $e^{-\lambda_1 r}$, a pressure wave associated with $e^{-\lambda_2 r}$ and a shear wave associated with $e^{-\lambda_3 r}$ and that the displacements are cylindrically symmetric around the direction of the force. Careful observation of eqns (15b) reveals that the contribution of shear wave associated with $e^{-\lambda_3 r}$ in the pressure due to point force is obviously zero. Also there is no shear wave present in the fields $\tilde{G}_{i4}$, $\tilde{G}_{44}$ radiated by a fluid point source. Both displacements and pressure due to fluid point source present a spherical symmetry centred on fluid point source. While pressure due to point force demonstrates cylindrical symmetry along the direction of the force, and an antisymmetry about the coordinate plane which passes through the force point and is perpendicular to the direction of the force (i.e. opposite sign of the values). These facts are consistent with the results of Burridge and Vargas (1979) and the prediction of Biot (1956a,b). In the corresponding frequency domain λ_1 and λ_2 are the wave numbers of the slow compressional waves and fast compressional waves, while λ_3 is the wave number of the shear waves.

We may easily verify that each column-vector in the fundamental solution matrix $\tilde{\mathbf{G}}$ possesses a unique singularity at the point $x = 0$ of the order $1/r$ for three-dimensional case.

Obviously each column of the fundamental solution matrix $\tilde{\mathbf{G}}$ satisfies system equations (6) and (7) or (12) and (13). Since the matrix $\tilde{\mathbf{G}}$ is unsymmetrical, as shown by eqns (15b) and (15c), its rows considered as vectors do not satisfy the equations, while Boutin *et al.* (1987) derived unsatisfactory results with $G_{i4} = G_{4i}$. Also, errors in Bonnet's (1987) work have been shown by Dominguez (1991, 1992).

Verification of the solutions

As a stringent test of the Laplace transform domain Green's function eqns (15), we now proceed to show that the fundamental solutions of elastodynamics in the Laplace transform domain and of steady-state poroelasticity can be recovered from eqn (15) by taking limits.

Limiting case 1: elastodynamics

For three-dimensional elastodynamics case, let κ approach infinity, ρ_f and m equal zero, eqns (16a–g) yield:

$$\frac{1}{\zeta} = 0, \tag{18a}$$

$$\alpha_1 = \alpha_2 = \alpha, \tag{18b}$$

$$\rho_1 = \rho, \tag{18c}$$

$$\Lambda^2 = \frac{\rho}{\lambda+2\mu}s^2, \tag{18d}$$

$$\lambda_1^2 = \frac{\rho}{\lambda+2\mu}s^2, \tag{18e}$$

$$\lambda_2^2 = 0, \tag{18f}$$

$$\lambda_3^2 = \frac{\rho}{\mu}s^2. \tag{18g}$$

Substituting eqns (18) into eqns (15), the fundamental solutions reduce to:

$$\tilde{G}_{ij}(x,s) = \frac{1}{4\pi\rho C_2^2}\left(a\delta_{ij} - b\frac{x_i x_j}{r^2}\right)\frac{1}{s} \tag{19a}$$

$$\tilde{G}_{4j}(x,s) = 0, \tag{19b}$$

$$\tilde{G}_{i4}(x,s) = 0, \tag{19c}$$

$$\tilde{G}_{44}(x,s) = 0, \tag{19d}$$

where:

$$a = \left(\frac{1}{r} + \frac{C_2}{sr^2} + \frac{C_2^2}{s^2r^3}\right)e^{-(sr/c_2)} - \left(\frac{C_2^2}{C_1^2}\right)\left(\frac{C_1}{sr^2} + \frac{C_1^2}{s^2r^3}\right)e^{-(sr/c_1)}, \tag{20a}$$

$$b = \left(\frac{1}{r} + \frac{3C_2}{sr^2} + \frac{3C_2^2}{s^2r^3}\right)e^{-(sr/c_2)} - \left(\frac{C_2^2}{C_1^2}\right)\left(\frac{1}{r} + \frac{3C_1}{sr^2} + \frac{3C_1^2}{s^2r^3}\right)e^{-(sr/c_1)}, \tag{20b}$$

$$C_1^2 = \frac{\lambda+2\mu}{\rho}, \tag{20c}$$

$$C_2^2 = \frac{\mu}{\rho}. \tag{20d}$$

C_1 and C_2 are propagation velocities of the pressure (dilatational) and shear (rotational) waves in the elastic solid, respectively. Equation (19a), after being multiplied by Laplace parameter s, agrees with Cruse and Rizzo (1968) which is the transform domain Green's function due to a Dirac delta time function.

Limiting case 2: steady-state poroelasticity

Now we validate the Green's function, by letting $t \to \infty$ i.e. $s \to 0$, to see if it reduces to the steady-state poroeleastic fundamental solution. Consider, first, the pressure due to a unit step fluid injection in an infinite medium at very long times. From eqns (16a–d), when $s \to 0$, we have λ_i $(i = 1, 2, 3) \to 0$. Thus $e^{-\lambda_i r}$ can be expressed in a power series to the first order in the form of:

$$e^{-\lambda_i r} = 1 - \lambda_i r. \tag{21}$$

Substituting eqn (21) in $\tilde{G}_{44}$ of eqn (15d), thus:

$$\begin{aligned}\lim_{t\to\infty} G_{44}(x,t) &= \lim_{s\to 0} s\tilde{G}_{44}(x,s)\\ &= \lim_{s\to 0}\frac{1}{4\pi r\zeta}\frac{1}{\lambda_1^2-\lambda_2^2}((\lambda_1^2-\Lambda^2)(1-\lambda_1 r)-(\lambda_2^2-\Lambda^2)(1-\lambda_2 r))\\ &= \lim_{s\to 0}\frac{1}{4\pi r\zeta} = \frac{1}{4\pi\kappa r}.\end{aligned} \tag{22}$$

This is, exactly, the potential flow Green's function. Next, consider the pressure due to a unit step force in the j-direction in an infinite medium at very long times. Instead of eqn (21), now $e^{-\lambda_i r}$ should be expressed in a power series to the second order of λ_i, i.e.

$$e^{-\lambda_i r} = 1 - \lambda_i r + \tfrac{1}{2}\lambda_i^2 r^2. \tag{23}$$

Similarly, from eqn (23) and $\tilde{G}_{i4}$ of eqns (15b–c), we directly obtain:

$$\begin{aligned}\lim_{t\to\infty} G_{i4}(x,t) &= \lim_{s\to 0} s\tilde{G}_{i4}(x,s)\\ &= \lim_{s\to 0}\frac{1}{4\pi(\lambda+2\mu)}\frac{\alpha_1}{\zeta}\frac{1}{\lambda_1^2-\lambda_2^2}\left(\tfrac{1}{2}(\lambda_1^2-\lambda_2^2)\frac{x_i}{r} - \tfrac{1}{2}x_i(\lambda_1^3-\lambda_2^3)\right).\end{aligned} \tag{24}$$

Neglecting terms of the third order in λ_i, i.e. λ_1^3 and λ_2^3 and noting that:

$$\lim_{s\to 0}\frac{\alpha_1}{\zeta} = \frac{\alpha}{\kappa}. \tag{25}$$

Equation (24) becomes:

$$\lim_{t\to\infty} G_{i4}(x,t) = \frac{1}{8\pi}\frac{\alpha}{\kappa(\lambda+2\mu)}\frac{x_i}{r}. \tag{26}$$

Equation (26) represents the steady-state displacement field caused by a constant unit fluid injection. Since:

$$\tilde{G}_{4i}(x,s) = s\tilde{G}_{i4}(x,s). \tag{27}$$

We directly have:

$$\lim_{t\to\infty} G_{4i}(x,t) = \lim_{s\to 0} s^2\tilde{G}_{i4}(x,s) = 0. \tag{28}$$

The above equation means that all fluid effects vanish at very long times due to a unit step force in the i-direction. Finally, we consider the displacement in the i-direction due to a unit step force in the j-direction at very long times. Similarly, substituting eqn (23) in $\tilde{G}_{ij}(x,s)$ of eqn (15a), one obtains:

$$\begin{aligned} s\tilde{G}_{ij}(x,s) = &-\frac{1}{4\pi}\frac{\Lambda^2-\lambda_2^2}{\lambda_2^2-\lambda_1^2}\frac{1}{\rho_1 s^2}\left(\left(\frac{3x_ix_j}{r^5}-\frac{\delta_{ij}}{r^3}\right)-\frac{1}{2}\left(\frac{x_ix_j}{r^3}-\frac{\delta_{ij}}{r}\right)\lambda_1^2+\frac{1}{2}\left(\frac{x_ix_j}{r^2}-\delta_{ij}\right)\lambda_1^3\right. \\ &\left.+\frac{1}{2}\frac{x_ix_j}{r}\lambda_1^4\right)+\frac{1}{4\pi}\frac{\Lambda^2-\lambda_1^2}{\lambda_2^2-\lambda_1^2}\frac{1}{\rho_1 s^2}\left(\left(\frac{3x_ix_j}{r^5}-\frac{\delta_{ij}}{r^3}\right)-\frac{1}{2}\left(\frac{x_ix_j}{r^3}-\frac{\delta_{ij}}{r}\right)\lambda_2^2\right. \\ &\left.+\frac{1}{2}\left(\frac{x_ix_j}{r^2}-\delta_{ij}\right)\lambda_2^3+\frac{1}{2}\frac{x_ix_j}{r}\lambda_2^4\right)-\frac{1}{4\pi}\frac{1}{\rho_1 s^2}\left(\left(\frac{3x_ix_j}{r^5}-\frac{\delta_{ij}}{r^3}\right)\right. \\ &\left.-\frac{1}{2}\left(\frac{x_ix_j}{r^3}-\frac{\delta_{ij}}{r}\right)\lambda_3^2+\frac{1}{2}\left(\frac{x_ix_j}{r^2}-\delta_{ij}\right)\lambda_3^3+\frac{1}{2}\frac{x_ix_j}{r}\lambda_3^4\right)+\frac{1}{4\pi\mu}\frac{1}{r}\delta_{ij}. \end{aligned} \tag{29}$$

Neglecting all the third and higher order of λ_i and with simple algebraic manipulation, eqn (29) becomes:

$$\begin{aligned} \lim_{t\to\infty} G_{ij}(x,t) &= \lim_{s\to 0} s\tilde{G}_{ij}(x,s) \\ &= \lim_{s\to 0}\left\{\frac{1}{4\pi}\frac{1}{\rho_1 s^2}\left(\frac{3x_ix_j}{r^5}-\frac{\delta_{ij}}{r^3}\right)0+\frac{1}{8\pi}\frac{1}{\rho_1 s^2}\left(\frac{x_ix_j}{r^3}-\frac{\delta_{ij}}{r}\right)(-\Lambda^2+\lambda_3^2)+\frac{1}{4\pi\mu}\frac{1}{r}\delta_{ij}\right\}. \end{aligned} \tag{30}$$

since:

$$\Lambda^2 = \frac{\rho_1 s^2}{\lambda+2\mu}$$

$$\lambda_3^2 = \frac{\rho_1 s^2}{\mu}.$$

Therefore

$$\begin{aligned} \lim_{t\to\infty} G_{ij}(x,t) &= \frac{1}{8\pi}\left(\frac{1}{\mu}-\frac{1}{\lambda+2\mu}\right)\frac{x_ix_j}{r^3}+\frac{1}{8\pi}\left(\frac{1}{\mu}+\frac{1}{\lambda+2\mu}\right)\frac{1}{r}\delta_{ij} \\ &= \frac{1}{16\pi r}\frac{1}{\mu(1-\nu)}\left[\frac{x_ix_j}{r^2}+(3-4\nu)\delta_{ij}\right]. \end{aligned} \tag{31}$$

Equation (31) is, simply, the Green's function for elastostatics.

This completes the proof.

TRANSIENT FUNDAMENTAL SOLUTION (LIMITING CASE-WAVEFRONT FORMULAS)

To return to the real time domain, we now need to invert the Laplace transformations that were used to obtain equations (15a–d). Since $\lambda_1, \lambda_2, \lambda_3$ are determined by eqns (16) which are exactly the same ones as in the two-dimensional case (Chen, 1993a), the approximate expressions for $\lambda_1, \lambda_2, \lambda_3$, and $\lambda_1^2 - \lambda_2^2$ for $s \to \infty$ i.e. $t \to 0$ can be borrowed directly from the two-dimensional case without any further work. They are expressed as follows:

$$\lambda_1^2 - \lambda_2^2 \approx \sqrt{a_3}(s+b)(s+c), \tag{32}$$

where b, c and a_3 are defined as:

$$\begin{aligned} b, c &= -\frac{1}{2}\left\{-\frac{a_1}{\kappa} \pm \sqrt{\left(\frac{a_1}{\kappa}\right)^2 - \frac{4a_2}{\kappa^2}}\right\}, \\ a_1 &= \frac{a_4}{2a_3}, \\ a_2 &= \frac{1}{2}\left(-\frac{1}{4}\frac{a_4^2}{a_3^2} + \frac{a_5}{a_3}\right), \\ a_3 &= \left(\frac{\rho+\alpha^2 m - 2\alpha\rho_f}{\lambda+2\mu} + \frac{m}{Q}\right)^2 + \frac{4(\rho_f^2 - \rho m)}{Q(\lambda+2\mu)}, \\ a_4 &= 2\left[\frac{-\rho+2\alpha^2 m - 2\alpha\rho_f}{Q(\lambda+2\mu)} + \frac{\alpha^2(\rho+\alpha^2 m - 2\alpha\rho_f)}{(\lambda+2\mu)^2} + \frac{m}{Q^2}\right], \\ a_5 &= \left(\frac{1}{Q} + \frac{\alpha^2}{\lambda+2\mu}\right)^2. \end{aligned} \tag{33}$$

For diffusive wave and compressive wave:

$$\lambda_{1,2} \approx \frac{1}{c_{d,p}}\sqrt{(s+\eta_{d,p})^2 - \xi_{d,p}^2}, \tag{34}$$

where

$$\begin{aligned} c_{d,p} &= \frac{\sqrt{2}}{\sqrt{a_6 \pm \sqrt{a_3}}}, \\ \eta_{d,p} &= \frac{1}{2}\frac{a_7 \pm \dfrac{a_4}{2\sqrt{a_3}}}{a_6 \pm \sqrt{a_3}}\frac{1}{\kappa}, \\ \xi_{d,p} &= \left\{(\eta_{d,p})^2 \mp \frac{\dfrac{1}{2}\left(-\dfrac{a_4^2}{4a_3^{3/2}} + \dfrac{a_5}{\sqrt{a_3}}\right)\dfrac{1}{\kappa^2}}{a_6 \pm \sqrt{a_3}}\right\}^{1/2}, \\ a_6 &= \frac{\rho+\alpha^2 m - 2\alpha\rho_f}{\lambda+2\mu} + \frac{m}{Q}, \\ a_7 &= \frac{1}{Q} + \frac{\alpha^2}{\lambda+2\mu}. \end{aligned} \tag{35}$$

For shear wave:

$$\lambda_3 = \frac{1}{c_s}\sqrt{(s+\eta_s)^2-\xi_s^2}, \tag{36}$$

where

$$c_s = \sqrt{\frac{\mu}{\rho}}\frac{1}{\sqrt{1-\frac{\rho_f^2}{\rho m}}},$$

$$\eta_s = \frac{1}{2\kappa m}\frac{\rho_f^2}{m\rho-\rho_f^2},$$

$$\xi_s = \left(\eta_s^2+\frac{\rho_f^2}{(m\rho-\rho_f^2)m^2\kappa^2}\right)^{1/2}. \tag{37}$$

In the above, c_d, c_p, c_s denote velocities of diffusive wave, pressure wave and shear wave, respectively, while η_d, η_p, η_s denote dissipation factors of the corresponding ones. As noted in Chen (1993a), this approximation is valid for large values of κs. Thus we come to the conclusion that for the dimensional form of the governing equations, an increase in permeability coefficient κ always results in an increase in dimensional time t, during which the time domain solution is valid. However, for the nondimensional case, where the nondimensional permeability coefficient κ is one, the approximate solution is valid only for small nondimensional time τ. With the asymptotic expansion being presented for $\lambda_1, \lambda_2, \lambda_3$ and $\lambda_1^2-\lambda_2^2$, we now proceed to solve for the time domain three-dimensional tensor Green's function by using analytical Laplace inversion.

Function G_{ij}

First we consider $\tilde{G}_{ij}$ i.e. eqn (15a) the displacement in x_i direction due to point force in x_j direction. Obviously eqn (15a) is not expressed in terms to which Laplace inversion could be directly applied. To examine the situation in a different way, we substitute eqns (16a–g) into (15a), and with some algebraic manipulation, we get:

$$\begin{aligned}\tilde{G}_{ij} = &\left\{-A_{ij}d_1\frac{1}{(s+a)(\lambda_1^2-\lambda_2^2)}-A_{ij}d_2\frac{1}{s(s+a)(\lambda_1^2-\lambda_2^2)}-A_{ij}d_3\frac{1}{s^2(s+a)(\lambda_1^2-\lambda_2^2)}\right.\\ &-A_{ij}d_4\frac{1}{s(\lambda_1^2-\lambda_2^2)}-A_{ij}d_{52}\frac{1}{s^3}-A_{ij}d_{62}\frac{1}{s^3(s+a)}-C_{ij}e_1\frac{1}{\lambda_1^2-\lambda_2^2}-C_{ij}e_2\frac{s}{\lambda_1^2-\lambda_2^2}\\ &\left.-C_{ij}e_{31}\frac{1}{s}\right\}e^{-\lambda_1 r}+\left\{-B_{ij}e_1\frac{1}{\lambda_1^2-\lambda_2^2}-B_{ij}e_2\frac{s}{\lambda_1^2-\lambda_2^2}-B_{ij}e_{31}\frac{1}{s}\right\}\frac{1}{\lambda_1}e^{-\lambda_1 r}\\ &+\left\{A_{ij}d_1\frac{1}{(s+a)(\lambda_1^2-\lambda_2^2)}+A_{ij}d_2\frac{1}{s(s+a)(\lambda_1^2-\lambda_2^2)}+A_{ij}d_3\frac{1}{s^2(s+a)(\lambda_1^2-\lambda_2^2)}\right.\\ &+A_{ij}d_4\frac{1}{s(\lambda_1^2-\lambda_2^2)}+A_{ij}d_{51}\frac{1}{s^3}+A_{ij}d_{61}\frac{1}{s^3(s+a)}+C_{ij}e_1\frac{1}{\lambda_1^2-\lambda_2^2}\\ &\left.+C_{ij}e_2\frac{s}{\lambda_1^2-\lambda_2^2}+C_{ij}e_{32}\frac{1}{s}\right\}e^{-\lambda_2 r}+\left\{B_{ij}e_1\frac{1}{\lambda_1^2-\lambda_2^2}+B_{ij}e_2\frac{s}{\lambda_1^2-\lambda_2^2}+B_{ij}e_{32}\frac{1}{s}\right\}\frac{1}{\lambda_2}e^{-\lambda_2 r}\\ &+\left\{(-A_{ij}b_1)\frac{1}{s^3(s+a)}+\left(-\frac{C_{ij}}{\mu}+D_{ij}\right)\frac{1}{s}+(-A_{ij}b_2)\frac{1}{s^3}\right\}e^{-\lambda_3 r}+\left(-B_{ij}\frac{1}{\mu}\frac{1}{s}\right)\frac{1}{\lambda_3}e^{-\lambda_3 r},\end{aligned} \tag{38}$$

where:

$$
\begin{aligned}
d_1 &= \frac{1}{2}\frac{1}{\rho m-\rho_f^2}\left(\frac{m^2}{Q}+\frac{\alpha^2m^2-2\alpha\rho_f m+\rho_f^2}{\lambda+2\mu}\right)\\
d_2 &= \frac{1}{2}\frac{1}{\rho m-\rho_f^2}\left(\frac{2m}{Q}+\frac{2\alpha^2m-2\alpha\rho_f}{\lambda+2\mu}\right)\frac{1}{\kappa}\\
d_3 &= \frac{1}{2}\frac{1}{\rho m-\rho_f^2}\left(\frac{1}{Q}+\frac{\alpha^2}{\lambda+2\mu}\right)\frac{1}{\kappa^2}\\
d_4 &= -\frac{1}{2}\frac{1}{\lambda+2\mu}\\
d_{51} &= \frac{1}{2}\frac{m}{\rho m-\rho_f^2}\\
d_{52} &= -\frac{1}{2}\frac{m}{\rho m-\rho_f^2}\\
d_{61} &= -\frac{1}{2}\frac{\rho_f^2}{\kappa(\rho m-\rho_f^2)^2}\\
d_{62} &= \frac{1}{2}\frac{\rho_f^2}{\kappa(\rho m-\rho_f^2)^2} \qquad \text{(39a)}\\
e_1 &= \frac{1}{2(\lambda+2\mu)\kappa}\left(\frac{1}{Q}-\frac{\alpha^2}{\lambda+2\mu}\right)\\
e_2 &= \frac{1}{2(\lambda+2\mu)}\left(\frac{m}{Q}+\frac{-\alpha^2m-\rho+2\alpha\rho_f}{\lambda+2u}\right)\\
e_{31} &= -\frac{1}{2(\lambda+2\mu)}\\
e_{32} &= \frac{1}{2(\lambda+2\mu)}\\
a &= \frac{1}{\kappa}\frac{\rho}{\rho m-\rho_f^2}\\
b_1 &= -\frac{\rho_f^2}{\kappa(\rho m-\rho_f^2)^2}\\
b_2 &= \frac{m}{\rho m-\rho_f^2}. \qquad \text{(39b)}
\end{aligned}
$$

Substituting eqns (32), which are asymptotic expressions for $\lambda_1^2-\lambda_2^2$, and keeping in mind eqns (34) and (36) we arrive at:

$$
\begin{aligned}
\tilde{G}_{ij} = \Bigg\{ & f_1\frac{1}{(s+a)(s+b)(s+c)}+f_2\frac{1}{s(s+a)(s+b)(s+c)}+f_3\frac{1}{s^2(s+a)(s+b)(s+c)}\\
&+f_4\frac{1}{(s+b)(s+c)}+f_5\frac{1}{s(s+b)(s+c)}+f_6\frac{1}{(s+c)}+f_7\frac{1}{s^3(s+a)}\\
&+f_8\frac{1}{s}+f_9\frac{1}{s^3}\Bigg\}e^{-\lambda_1 r}+\left\{f_{10}\frac{1}{(s+b)(s+c)}+f_{11}\frac{1}{s+c}+f_{12}\frac{1}{s}\right\}\frac{1}{\lambda_1}e^{-\lambda_1 r}
\end{aligned}
$$

$$+\left\{g_1\frac{1}{(s+a)(s+b)(s+c)}+g_2\frac{1}{s(s+a)(s+b)(s+c)}+g_3\frac{1}{s^2(s+a)(s+b)(s+c)}\right.$$

$$+g_4\frac{1}{(s+b)(s+c)}+g_5\frac{1}{s(s+b)(s+c)}+g_6\frac{1}{(s+c)}+g_7\frac{1}{s^3(s+a)}$$

$$\left.+g_8\frac{1}{s}+g_9\frac{1}{s^3}\right\}e^{-\lambda_2 r}+\left\{g_{10}\frac{1}{(s+b)(s+c)}+g_{11}\frac{1}{s+c}+g_{12}\frac{1}{s}\right\}\frac{1}{\lambda_2}e^{-\lambda_2 r}$$

$$+\left\{h_1\frac{1}{s^3(s+a)}+h_2\frac{1}{s}+h_3\frac{1}{s^3}\right\}e^{-\lambda_3 r}+\left\{h_4\frac{1}{s}\right\}\frac{1}{\lambda_3}e^{-\lambda_3 r} \tag{40}$$

where:

$$\begin{aligned}
f_1 &= -\frac{A_{ij}d_1}{\sqrt{a_3}}, & g_1 &= \frac{A_{ij}d_1}{\sqrt{a_3}}, & h_1 &= -A_{ij}b_1,\\
f_2 &= -\frac{A_{ij}d_2}{\sqrt{a_3}}, & g_2 &= \frac{A_{ij}d_2}{\sqrt{a_3}}, & h_2 &= -\frac{C_{ij}}{\mu}+D_{ij},\\
f_3 &= -\frac{A_{ij}d_3}{\sqrt{a_3}}, & g_3 &= \frac{A_{ij}d_3}{\sqrt{a_3}}, & h_3 &= -A_{ij}b_2,\\
f_4 &= \frac{C_{ij}e_2b-C_{ij}e_1}{\sqrt{a_3}}, & g_4 &= \frac{-C_{ij}e_2b+C_{ij}e_1}{\sqrt{a_3}}, & h_4 &= -\frac{B_{ij}}{\mu}.\\
f_5 &= -\frac{A_{ij}d_4}{\sqrt{a_3}}, & g_5 &= \frac{A_{ij}d_4}{\sqrt{a_3}},\\
f_6 &= -\frac{C_{ij}e_2}{\sqrt{a_3}}, & g_6 &= \frac{C_{ij}e_2}{\sqrt{a_3}},\\
f_7 &= -A_{ij}d_{62}, & g_7 &= A_{ij}d_{61},\\
f_8 &= -C_{ij}e_{31}, & g_8 &= C_{ij}e_{32},\\
f_9 &= -A_{ij}d_{52}, & g_9 &= A_{ij}d_{51},\\
f_{10} &= \frac{-B_{ij}e_1+B_{ij}e_2b}{\sqrt{a_3}}, & g_{10} &= \frac{B_{ij}e_1-B_{ij}e_2b}{\sqrt{a_3}},\\
f_{11} &= -\frac{B_{ij}e_2}{\sqrt{a_3}}, & g_{11} &= \frac{B_{ij}e_2}{\sqrt{a_3}},\\
f_{12} &= -B_{ij}e_{31}, & g_{12} &= B_{ij}e_{32}.
\end{aligned} \tag{41}$$

To invert eqn (40), we first observe the following formula (Abramowitz and Stegun, 1965):

$$L^{-1}\{e^{-(r/c)\sqrt{s^2+2\xi s}}\}=e^{-\xi t}\left[\frac{\xi\frac{r}{c}}{\sqrt{t^2-\frac{r^2}{c^2}}}I_1\left(\xi\sqrt{t^2-\frac{r^2}{c^2}}\right)+\delta\left(t-\frac{r}{c}\right)\right]H\left(t-\frac{r}{c}\right), \tag{42}$$

also

$$L^{-1}\left\{\frac{1}{\sqrt{s^2-a^2}}e^{-b\sqrt{s^2-a^2}}\right\}=I_0(a\sqrt{t^2-b^2})H(t-b), \tag{43}$$

where I_0, I_1 is the modified Bessel function for the first kind of order zero and order one respectively. $H(t)$ is the Heaviside step function and $\delta(t)$ is the Dirac delta function.

Equation (40) can now easily be inverted by the use of eqns (42) and (43) and employing convolution theorem and other properties of the inverse transforms, to give:

$$
\begin{aligned}
G_{ij} = & \int_{r/c_d}^{t} [P_{11}e^{-b(t-\tau)} + P_{12}e^{-c(t-\tau)} + P_{13}]e^{-\eta_d\tau} I_0\left(\xi_d\sqrt{\tau^2 - r^2/c_d^2}\right) d\tau H(t - r/c_d) \\
& + \int_{r/c_d}^{t} [P_{21}e^{-a(t-\tau)} + P_{22}e^{-b(t-\tau)} + P_{23}e^{-c(t-\tau)} + P_{24} + P_{25}(t-\tau) + P_{26}(t-\tau)^2] \\
& \times e^{-\eta_d\tau} \frac{\xi_d r/c_d}{\sqrt{\tau^2 - r/c_d^2}} I_1\left(\xi_d\sqrt{\tau^2 - r^2/c_d^2}\right) d\tau H(t - r/c_d) + \left[P_{21}e^{-a(t-r/c_d)} + P_{22}e^{-b(t-r/c_d)} \right. \\
& \left. + P_{23}e^{-c(t-r/c_d)} + P_{24} + P_{25}\left(t - \frac{r}{c_d}\right) + P_{26}(t - r/c_d)^2 \right] e^{-\eta_d r/c_d} H(t - r/c_d) \\
& + \int_{r/c_p}^{t} [P_{31}e^{-b(t-\tau)} + P_{32}e^{-c(t-\tau)} + P_{33}]e^{-\eta_p\tau} I_0\left(\xi_p\sqrt{\tau^2 - r^2/c_p^2}\right) d\tau H(t - r/c_p) \\
& + \int_{r/c_p}^{t} [P_{41}e^{-a(t-\tau)} + P_{42}e^{-b(t-\tau)} + P_{43}e^{-c(t-\tau)} + P_{44} + P_{45}(t-\tau) + P_{46}(t-\tau)^2] \\
& \times e^{-\eta_p\tau} \frac{\xi_p r/c_p}{\sqrt{\tau^2 - r/c_p^2}} I_1\left(\xi_p\sqrt{\tau^2 - r^2/c_p^2}\right) d\tau H(t - r/c_p) + \left[P_{41}e^{-a(t-r/c_p)} + P_{42}e^{-b(t-r/c_p)} \right. \\
& \left. + P_{43}e^{-c(t-r/c_p)} + P_{44} + P_{45}\left(t - \frac{r}{c_p}\right) + P_{46}(t - r/c_p)^2 \right] e^{-\eta_p r/c_p} H(t - r/c_p) \\
& + \int_{r/c_s}^{t} P_{51}e^{-\eta_s\tau} I_0\left(\xi_s\sqrt{\tau^2 - r^2/c_s^2}\right) d\tau H(t - r/c_s) + \int_{r/c_s}^{t} [P_{61}e^{-a(t-\tau)} + P_{62} \\
& + P_{63}(t-\tau) + P_{64}(t-\tau)^2]e^{-\eta_s\tau} \frac{\xi_s r/c_s}{\sqrt{\tau^2 - r^2/c_s^2}} I_1\left(\xi_s\sqrt{\tau^2 - r^2/c_s^2}\right) d\tau H(t - r/c_s) \\
& + \left[P_{61}e^{-a(t-r/c_s)} + P_{62} + P_{63}\left(t - \frac{r}{c_s}\right) + P_{64}(t - r/c_s)^2 \right] e^{-\eta_s r/c_s} H(t - r/c_s),
\end{aligned}
\tag{44}
$$

where:

$$P_{11} = \frac{f_{10}}{c-b} c_d$$

$$P_{12} = \left(\frac{-f_{10}}{c-b} + f_{11}\right) c_d$$

$$P_{13} = f_{12} c_d$$

$$P_{21} = \frac{f_1}{(b-a)(c-a)} - \frac{f_2}{a(b-a)(c-a)} + \frac{f_3}{a^2(b-a)(c-a)} - \frac{f_7}{a^3}$$

$$P_{22} = \frac{f_1}{(a-b)(c-b)} - \frac{f_2}{b(a-b)(c-b)} + \frac{f_3}{b^2(a-b)(c-b)} + \frac{f_4}{c-b} + \frac{f_5}{b(b-c)}$$

$$P_{23} = \frac{f_1}{(a-c)(b-c)} - \frac{f_2}{c(a-c)(b-c)} + \frac{f_3}{c^2(a-c)(b-c)} - \frac{f_4}{c-b} - \frac{f_5}{c(b-c)} + f_6$$

$$P_{24} = \frac{f_2}{abc} - \frac{(ab+bc+ca)f_3}{(abc)^2} + \frac{f_5}{bc} + \frac{f_7}{a^3} + f_8$$

$$P_{25} = \frac{f_3}{abc} - \frac{f_7}{a^2}$$

$$P_{26} = \frac{f_7}{2a} + \frac{f_9}{2} \tag{45a}$$

$$P_{31} = \frac{g_{10}}{c-b} c_p$$

$$P_{32} = \left(\frac{-g_{10}}{c-b} + g_{11}\right) c_p$$

$$P_{33} = g_{12} c_p$$

$$P_{41} = \frac{g_1}{(b-a)(c-a)} - \frac{g_2}{a(b-c)(c-a)} + \frac{g_3}{a^2(b-a)(c-a)} - \frac{g_7}{a^3}$$

$$P_{42} = \frac{g_1}{(a-b)(c-b)} - \frac{g_2}{b(a-b)(c-b)} + \frac{g_3}{b^2(a-b)(c-b)} + \frac{g_4}{c-b} + \frac{g_5}{b(b-c)}$$

$$P_{43} = \frac{g_1}{(a-c)(b-c)} - \frac{g_2}{c(a-c)(b-c)} + \frac{g_3}{c^2(a-c)(b-c)} - \frac{g_4}{c-b} - \frac{g_5}{c(b-c)} + g_6$$

$$P_{44} = \frac{g_2}{abc} - \frac{(ab+bc+ca)g_3}{(abc)^2} + \frac{g_5}{bc} + \frac{g_7}{a^3} + g_8$$

$$P_{45} = \frac{g_3}{abc} - \frac{g_7}{a^2}$$

$$P_{46} = \frac{g_7}{2a} + \frac{g_9}{2}$$

$$P_{51} = h_4 c_s$$

$$P_{61} = -\frac{h_1}{a^3}$$

$$P_{62} = \frac{h_1}{a^3} + h_2$$

$$P_{63} = -\frac{h_1}{a^2}$$

$$P_{64} = \frac{h_1}{2a} + \frac{h_3}{2}. \tag{45b}$$

The physical interpretation of the various parts of eqn (44) is straightforward. The first three components represent that part of the solution which results from slow compressional wave (diffusive wave or P_2 wave) generated at the source, the next three components are that part which results from the fast compressional wave (pressure wave or P_1 wave) generated at the source, while the last three components are due to the shear wave (equivoluminal wave) generated at the source. The Heaviside step functions $H(t-r/c_p)$, $H(t-r/c_d)$ and $H(t-r/c_s)$ represent three wave fronts travelling in the r-direction with constant velocities of c_p (P_1 wave), c_d (P_2 wave), and c_s (S wave) and arrive at the time r/c_p, r/c_d and r/c_s, respectively. The attenuation of the wave of type P_1, P_2 and S depends upon the magnitude of η_p, η_d and η_s, respectively.

Function G_{4i}

We now proceed to solve the eqn (15b). To simplify the analysis, we note that by substituting eqns (16a–g) and rearranging terms, eqn (15b) can be rewritten as:

$$
\begin{aligned}
4\pi(\lambda+2\mu)\tilde{G}_{4i} = & -\frac{x_i}{r^3}\left(d_1\frac{1}{\lambda_1^2-\lambda_2^2}+d_2\frac{s}{\lambda_1^2-\lambda_2^2}\right)e^{-\lambda_1 r} \\
& -\frac{x_i}{r^2}\left(e_{11}s+e_{21}+e_3\frac{s}{\lambda_1^2-\lambda_2^2}+e_4\frac{s^2}{\lambda_1^2-\lambda_2^2}+e_5\frac{s^3}{\lambda_1^2-\lambda_2^2}\right)\frac{1}{\lambda_1}e^{-\lambda_1 r} \\
& +\frac{x_i}{r^3}\left(d_1\frac{1}{\lambda_1^2-\lambda_2^2}+d_2\frac{s}{\lambda_1^2-\lambda_2^2}\right)e^{-\lambda_2 r} \\
& +\frac{x_i}{r^2}\left(e_{12}s+e_{22}+e_3\frac{s}{\lambda_1^2-\lambda_2^2}+e_4\frac{s^2}{\lambda_1^2-\lambda_2^2}+e_5\frac{s^3}{\lambda_1^2-\lambda_2^2}\right)\frac{1}{\lambda_2}e^{-\lambda_2 r}.
\end{aligned}
\tag{46}
$$

where

$$
\begin{aligned}
d_1 &= \frac{\alpha}{\kappa} \\
d_2 &= \alpha m-\rho_f \\
e_{11} &= \tfrac{1}{2}(\alpha m-\rho_f) \\
e_{12} &= -\tfrac{1}{2}(\alpha m-\rho_f) \\
e_{21} &= \frac{\alpha}{2\kappa} \\
e_{22} &= -\frac{\alpha}{2\kappa} \\
e_3 &= \frac{1}{2}\left(\frac{1}{Q}+\frac{\alpha^2}{\lambda+2\mu}\right)\frac{\alpha}{\kappa^2} \\
e_4 &= \frac{1}{2}\left[\frac{2\alpha m-\rho_f}{Q}+\frac{2\alpha^3 m-3\alpha^2\rho_f+\alpha\rho}{\lambda+2\mu}\right]\frac{1}{\kappa} \\
e_5 &= \frac{1}{2}\left(\frac{m}{Q}+\frac{\alpha^2 m-2\alpha\rho_f+\rho}{\lambda+2\mu}\right)(\alpha m-\rho_f).
\end{aligned}
\tag{47}
$$

By using eqn (32) i.e. $\lambda_1^2-\lambda_2^2=\sqrt{a_3}(s+b)(s+c)$, eqn (46) leads to:

$$
\begin{aligned}
\tilde{G}_{4i} = {} & f_1\frac{1}{(s+b)(s+c)}e^{-\lambda_1 r}+f_2\frac{1}{(s+c)}e^{-\lambda_1 r}+f_3 s\frac{1}{\lambda_1}e^{-\lambda_1 r}+f_4\frac{1}{\lambda_1}e^{-\lambda_1 r}+f_5\frac{1}{s+c}\frac{1}{\lambda_1}e^{-\lambda_1 r} \\
& +f_6\frac{1}{(s+b)(s+c)}\frac{1}{\lambda_1}e^{-\lambda_1 r}+g_1\frac{1}{(s+b)(s+c)}e^{-\lambda_2 r}+g_2\frac{1}{(s+c)}e^{-\lambda_2 r}+g_3 s\frac{1}{\lambda_2}e^{-\lambda_2 r} \\
& +g_4\frac{1}{\lambda_2}e^{-\lambda_2 r}+g_5\frac{1}{s+c}\frac{1}{\lambda_2}e^{-\lambda_2 r}+g_6\frac{1}{(s+b)(s+c)}\frac{1}{\lambda_2}e^{-\lambda_2 r},
\end{aligned}
\tag{48}
$$

where:

$$f_1 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^3}\frac{d_1-bd_2}{\sqrt{a_3}}$$
$$f_2 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^3}\frac{d_2}{\sqrt{a_3}}$$
$$f_3 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(e_{11}+\frac{e_5}{\sqrt{a_3}}\right)$$
$$f_4 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(e_{21}+\frac{e_4-e_5(b+c)}{\sqrt{a_3}}\right)$$
$$f_5 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(\frac{e_3-e_4(b+c)+e_5(b^2+bc+c^2)}{\sqrt{a_3}}\right)$$
$$f_6 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(\frac{-be_3+b^2e_4-b^3e_5}{\sqrt{a_3}}\right), \tag{49a}$$

$$g_1 = \frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^3}\frac{d_1-bd_2}{\sqrt{a_3}}$$
$$g_2 = \frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^3}\frac{d_2}{\sqrt{a_3}}$$
$$g_3 = \frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(e_{12}+\frac{e_5}{\sqrt{a_3}}\right)$$
$$g_4 = \frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(e_{22}+\frac{e_4-e_5(b+c)}{\sqrt{a_3}}\right)$$
$$g_5 = \frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(\frac{e_3-e_4(b+c)+e_5(b^2+bc+c^2)}{\sqrt{a_3}}\right)$$
$$g_6 = \frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(\frac{-be_3+b^2e_4-b^3e_5}{\sqrt{a_3}}\right). \tag{49b}$$

Inverting the above equation we find after much manipulation that the required solution is given by:

$$\begin{aligned}G_{4i} &= P_{11}e^{-\eta_d r/c_d}\delta(t-r/c_d)+P_{12}e^{-\eta_d t}I_0\left(\xi_d\sqrt{t^2-r^2/c_d^2}\right)H(t-r/c_d)\\
&+P_{13}e^{-\eta_d t}I_1\left(\xi_d\sqrt{t^2-r^2/c_d^2}\right)\frac{t}{\sqrt{t^2-r^2/c_d^2}}H(t-r/c_d)+\int_{r/c_d}^{t}[P_{21}e^{-b(t-\tau)}\\
&+P_{22}e^{-c(t-\tau)}]e^{-\eta_d\tau}\frac{\xi_d r/c_d}{\sqrt{\tau^2-r^2/c_d^2}}I_1\left(\xi_d\sqrt{\tau^2-r^2/c_d^2}\right)d\tau H(t-r/c_d)\\
&+(P_{21}e^{-b(t-r/c_d)}+P_{22}e^{-c(t-r/c_d)})e^{-\eta_d r/c_d}H(t-r/c_d)+\int_{r/c_d}^{t}[P_{31}e^{-b(t-\tau)}\\
&+P_{32}e^{-c(t-\tau)}]e^{-\eta_d t}I_0\left(\xi_d\sqrt{\tau^2-r^2/c_d^2}\right)d\tau H(t-r/c_d)+P_{41}e^{-\eta_p r/c_p}\delta(t-r/c_p)\\
&+P_{42}e^{-\eta_p t}I_0\left(\xi_p\sqrt{t^2-r^2/c_p^2}\right)H(t-r/c_p)+P_{43}e^{-\eta_p t}I_1\left(\xi_p\sqrt{t^2-r^2/c_p^2}\right)\end{aligned}$$

$$\times \frac{t}{\sqrt{t^2-r^2/c_p^2}} H(t-r/c_p)+\int_{r/c_p}^{t}[P_{51}e^{-b(t-\tau)}+P_{52}e^{-c(t-\tau)}]e^{-\eta_p\tau}\frac{\xi_p r/c_p}{\sqrt{\tau^2-r^2/c_p^2}}$$

$$\times I_1(\xi_p\sqrt{\tau^2-r^2/c_p^2})\,d\tau H(t-r/c_p)+(P_{51}e^{-b(t-r/c_p)}+P_{52}e^{-c(t-r/c_p)})e^{-\eta_p r/c_p}H(t-r/c_p)$$

$$+\int_{r/c_p}^{t}[P_{61}e^{-b(t-\tau)}+P_{62}e^{-c(t-\tau)}]e^{-\eta_p\tau}I_0(\xi_p\sqrt{\tau^2-r^2/c_p^2})\,d\tau H(t-r/c_p), \tag{50}$$

where:

$$\begin{aligned}
P_{11} &= f_3 c_d \\
P_{12} &= (f_4 - f_3\eta_d)c_d \\
P_{13} &= f_3 c_d \xi_d \\
P_{21} &= \frac{f_1}{c-b} \\
P_{22} &= -\frac{f_1}{c-b}+f_2 \\
P_{31} &= \frac{f_6 c_d}{c-b} \\
P_{32} &= \left(-\frac{f_6}{c-b}+f_5\right)c_d \\
P_{41} &= g_3 c_p \\
P_{42} &= (g_4 - g_3\eta_p)c_p \\
P_{43} &= g_3 c_p \xi_p \\
P_{51} &= \frac{g_1}{c-b} \\
P_{52} &= -\frac{g_1}{c-b}+g_2 \\
P_{61} &= \frac{g_6 c_p}{c-b} \\
P_{62} &= \left(-\frac{g_6}{c-b}+g_5\right)c_p.
\end{aligned} \tag{51}$$

Comparison of eqn (50) with eqn (44) demonstrates that the contribution of s-waves in the pressure due to point force is obviously zero. Additionally eqn (50) reveals that there are two pulses, which take the form of delta function and correspond to the arrival of pressure wave and diffusive wave, appearing in excessive pore fluid pressure due to point force.

Function G_{i4}

Now turn to $\tilde{G}_{i4}$, since:

$$\tilde{G}_{i4} = \tilde{G}_{4i}\frac{1}{s}. \tag{52}$$

For this situation, using eqn (48), we obtain:

$$\tilde{G}_{i4} = f_1 \frac{1}{s(s+b)(s+c)} e^{-\lambda_1 r} + f_2 \frac{1}{s(s+c)} e^{-\lambda_1 r} + f_3 \frac{1}{\lambda_1} e^{-\lambda_1 r} + f_4 \frac{1}{s} \frac{1}{\lambda_1} e^{-\lambda_1 r}$$
$$+ f_5 \frac{1}{s(s+c)} \frac{1}{\lambda_1} e^{-\lambda_1 r} + f_6 \frac{1}{s(s+b)(s+c)} \frac{1}{\lambda_1} e^{-\lambda_1 r} + g_1 \frac{1}{s(s+b)(s+c)} e^{-\lambda_2 r} + g_2 \frac{1}{s(s+c)} e^{-\lambda_2 r}$$
$$+ g_3 \frac{1}{\lambda_2} e^{-\lambda_2 r} + g_4 \frac{1}{s} \frac{1}{\lambda_2} e^{-\lambda_2 r} + g_5 \frac{1}{s(s+c)} \frac{1}{\lambda_2} e^{-\lambda_2 r} + g_6 \frac{1}{s(s+b)(s+c)} \frac{1}{\lambda_2} e^{-\lambda_2 r}, \quad (53)$$

where $f_1, f_2, \ldots, f_6, g_1, g_2, \ldots, g_6$ have the same values as in eqns (49). Taking the Laplace inversion of eqn (53), one obtains:

$$G_{i4} = P_{11} e^{-\eta_d t} I_0\left(\xi_d \sqrt{t^2 - r^2/c_d^2}\right) H(t - r/c_d) + \int_{r/c_d}^{t} [P_{21} e^{-b(t-\tau)} + P_{22} e^{-c(t-\tau)} + P_{23}]$$
$$\times e^{-\eta_d \tau} \frac{\xi_d r/c_d}{\sqrt{\tau^2 - r^2/c_d^2}} I_1\left(\xi_d \sqrt{\tau^2 - r^2/c_d^2}\right) d\tau H(t - r/c_d) + (P_{21} e^{-b(t - r/c_d)} + P_{22} e^{-c(t - r/c_d)} + P_{23})$$
$$\times e^{-\eta_d r/c_d} H(t - r/c_d) + \int_{r/c_d}^{t} [P_{31} e^{-b(t-\tau)} + P_{32} e^{-c(t-\tau)} + P_{33}] e^{-\eta_d \tau}$$
$$\times I_0\left(\xi_d \sqrt{\tau^2 - r^2/c_d^2}\right) d\tau H(t - r/c_d) + P_{41} e^{-\eta_p t} I_0\left(\xi_p \sqrt{t^2 - r^2/c_p^2}\right) H(t - r/c_p)$$
$$+ \int_{r/c_p}^{t} [P_{51} e^{-b(t-\tau)} + P_{52} e^{-c(t-\tau)} + P_{53}] e^{-\eta_p \tau} \frac{\xi_p r/c_p}{\sqrt{\tau^2 - r^2/c_p^2}} I_1\left(\xi_p \sqrt{\tau^2 - r^2/c_p^2}\right)$$
$$\times d\tau H(t - r/c_p) + (P_{51} e^{-b(t - r/c_p)} + P_{52} e^{-c(t - r/c_p)} + P_{53}) e^{-\eta_p r/c_p} H(t - r/c_p)$$
$$+ \int_{r/c_p}^{t} [P_{61} e^{-b(t-\tau)} + P_{62} e^{-c(t-\tau)} + P_{63}] e^{-\eta_p \tau} I_0\left(\xi_p \sqrt{\tau^2 - r^2/c_p^2}\right) d\tau H(t - r/c_p) \quad (54)$$

where:

$$P_{11} = f_3 c_d$$
$$P_{21} = \frac{f_1}{b(b-c)}$$
$$P_{22} = -\frac{f_1}{c(b-c)} - \frac{f_2}{c}$$
$$P_{23} = \frac{f_1}{bc} + \frac{f_2}{c}$$
$$P_{31} = \frac{f_6 c_d}{b(b-c)}$$
$$P_{32} = \left(-\frac{f_6}{c(b-c)} - \frac{f_5}{c}\right) c_d$$
$$P_{33} = \left(\frac{f_6}{bc} + \frac{f_5}{c} + f_4\right) c_d$$
$$P_{41} = g_3 c_p$$
$$P_{51} = \frac{g_1}{b(b-c)}$$
$$P_{52} = -\frac{g_1}{c(b-c)} - \frac{g_2}{c}$$

$$P_{53} = \frac{g_1}{bc} + \frac{g_2}{c}$$

$$P_{61} = \frac{g_6 c_p}{b(b-c)}$$

$$P_{62} = \left(-\frac{g_6}{c(b-c)} - \frac{g_5}{c}\right)c_p$$

$$P_{63} = \left(\frac{g_6}{bc} + \frac{g_5}{c} + g_4\right)c_p. \tag{55}$$

Like eqn (50), in the displacement field due to point fluid injection, there are only two dilatational waves. Since the displacement is finite, no pulse term appears in eqn (54).

Function G_{44}

It is possible to write the eqn (15d) in the alternative form:

$$8\pi r\tilde{G}_{44} = me^{-\lambda_1 r} + \frac{1}{\kappa}\frac{1}{s}e^{-\lambda_1 r} + d_1\frac{1}{\lambda_1^2-\lambda_2^2}e^{-\lambda_1 r} + d_2\frac{s}{\lambda_1^2-\lambda_2^2}e^{-\lambda_1 r} + d_3\frac{s^2}{\lambda_1^2-\lambda_2^2}e^{-\lambda_1 r}$$
$$+me^{-\lambda_2 r} + \frac{1}{\kappa}\frac{1}{s}e^{-\lambda_2 r} - d_1\frac{1}{\lambda_1^2-\lambda_2^2}e^{-\lambda_2 r} - d_2\frac{s}{\lambda_1^2-\lambda_2^2}e^{-\lambda_2 r} - d_3\frac{s^2}{\lambda_1^2-\lambda_2^2}e^{-\lambda_2 r}. \tag{56}$$

where:

$$d_1 = \frac{1}{\kappa^2}\left(\frac{1}{Q} + \frac{\alpha^2}{\lambda+2\mu}\right)$$

$$d_2 = \frac{1}{\kappa}\left(\frac{2m}{Q} + \frac{2\alpha^2 m - 2\alpha\rho_f - \rho}{\lambda+2\mu}\right)$$

$$d_3 = \frac{m^2}{Q} + \frac{\alpha^2 m^2 - m(\rho+2\alpha\rho_f) + 2\rho_f^2}{\lambda+2\mu}. \tag{57}$$

Putting $\lambda_1^2-\lambda_2^2 = \sqrt{a_3}(s+b)(s+c)$, in eqn (56) we therefore find that:

$$\tilde{G}_{44} = \left\{f_1 + f_2\frac{1}{s} + f_3\frac{1}{s+c} + f_4\frac{1}{(s+b)(s+c)}\right\}e^{-\lambda_1 r}$$
$$+\left\{g_1 + g_2\frac{1}{s} + g_3\frac{1}{s+c} + g_4\frac{1}{(s+b)(s+c)}\right\}e^{-\lambda_2 r} \tag{58}$$

where:

$$f_1 = \frac{1}{8\pi r}\left(\frac{d_3}{\sqrt{a_3}} + m\right)$$

$$f_2 = \frac{1}{8\pi r}\frac{1}{\kappa}$$

$$f_3 = \frac{1}{8\pi r}\frac{d_2 - d_3(b+c)}{\sqrt{a_3}}$$

$$f_4 = \frac{1}{8\pi r}\frac{d_1 - d_2 b + d_3 b^2}{\sqrt{a_3}}$$

$$g_1 = -\frac{1}{8\pi r}\left(\frac{d_3}{\sqrt{a_3}} - m\right)$$

$$g_2 = \frac{1}{8\pi r}\frac{1}{\kappa}$$

$$g_3 = -\frac{1}{8\pi r}\frac{d_2 - d_3(b+c)}{\sqrt{a_3}}$$

$$g_4 = -\frac{1}{8\pi r}\frac{d_1 - d_2 b + d_3 b^2}{\sqrt{a_3}} \tag{59}$$

Applying the Laplace inversion theorem, the required solution is easily found to be:

$$\begin{aligned} G_{44} = {} & P_{11}e^{-\eta_d t}\delta(t - r/c_d) + P_{12}e^{-\eta_d t}\frac{1}{\sqrt{t^2 - r^2/c_d^2}} I_1(\xi_d\sqrt{t^2 - r^2/c_d^2})H(t - r/c_d) \\ & + \int_{r/c_d}^{t} (P_{13}e^{-b(t-\tau)} + P_{14}e^{-c(t-\tau)} + P_{15})e^{-\eta_d \tau}\frac{\xi_d r/c_d}{\sqrt{\tau^2 - r^2/c_d^2}} \\ & \times I_1(\xi_d\sqrt{\tau^2 - r^2/c_d^2})\,d\tau H(t - r/c_d) + (P_{13}e^{-b(t-r/c_d)} + P_{14}e^{-c(t-r/c_d)} + P_{15}) \\ & \times e^{-\eta_d r/c_d}H(t - r/c_d) + P_{21}e^{-\eta_p t}\delta(t - r/c_p) + P_{22}e^{-\eta_p t}\frac{1}{\sqrt{t^2 - r^2/c_p^2}} \\ & \times I_1(\xi_p\sqrt{t^2 - r^2/c_p^2})H(t - r/c_p) + \int_{r/c_p}^{t} (P_{23}e^{-b(t-\tau)} + P_{24}e^{-c(t-\tau)} + P_{25})e^{-\eta_p \tau} \\ & \times \frac{\xi_p r/c_p}{\sqrt{\tau^2 - r^2/c_p^2}} I_1(\xi_p\sqrt{\tau^2 - r^2/c_p^2})\,d\tau H(t - r/c_p) \\ & + (P_{23}e^{-b(t-r/c_p)} + P_{24}e^{-c(t-r/c_p)} + P_{25})e^{-\eta_p r/c_p}H(t - r/c_p), \end{aligned} \tag{60}$$

where

$$P_{11} = f_1$$

$$P_{12} = f_1\xi_d\frac{r}{c_d}$$

$$P_{13} = \frac{f_4}{c - b}$$

$$P_{14} = f_3 - \frac{f_4}{c - b}$$

$$P_{15} = f_2$$

$$P_{21} = g_1$$

$$P_{22} = g_1\xi_p\frac{r}{c_p}$$

$$P_{23} = \frac{g_4}{c - b}$$

$$P_{24} = g_3 - \frac{g_4}{c - b}$$

$$P_{25} = g_2. \tag{61}$$

Both eqn (54) and eqn (60) reveal that there is no shear wave in the field radiated by a fluid injection, while both eqns (50) and (60) show that in the pressure field there are two pulse terms due to the arrival of P_1 wave and P_2 wave.

TRANSIENT FUNDAMENTAL SOLUTION (GENERAL CASE-APPROXIMATE GLOBAL FORMULAS)

In the previous section λ_1, λ_2, λ_3 and $\lambda_1^2-\lambda_2^2$ are approximated in powers of $1/s$ (or $1/\kappa s$) to a second order polynomial. Thus the transient fundamental solutions obtained constitute a good approximation to the exact results for short time or large permeability. In order to find the approximate solutions valid for the general case, an alternative approach is pursued. The inspection of eqns (16) shows that $\lambda_1, \lambda_2, \lambda_3$ and $\lambda_1^2-\lambda_2^2$ are defined by the same equations as in the two-dimensional case (Chen, 1993a), thus an approximation in terms of small nondimensional functional parameter i.e. $|s/((1/\kappa m)+s)| < 1$, which holds for all s and as s decreases (i.e. t increases) $|s/((1/\kappa m)+s)| \ll 1$ dominates, can be introduced again following Chen (1992, 1993a). We arrive at:

$$\lambda_1^2-\lambda_2^2 \approx cs(s+b), \tag{62}$$

where

$$c = \frac{m(\lambda+2\mu+\alpha^2 Q)}{Q(\lambda+2\mu)} - \frac{\rho+2\alpha\rho_f}{\lambda+2\mu+\alpha^2 Q} + \frac{Q\alpha^2(\rho-2\alpha\rho_f)}{(\lambda+2\mu)(\lambda+2\mu+\alpha^2 Q)}, \tag{63a}$$

$$cb = \frac{(\lambda+2\mu+\alpha^2 Q)}{Q(\lambda+2\mu)\kappa}, \tag{63b}$$

by the same way and by use of radiation condition, we obtain, from eqns (16):

$$\lambda_{1,2} = \frac{1}{c_{d,p}}\sqrt{(s+\eta_{d,p})^2-\xi_{d,p}^2} \tag{64}$$

where:

$$c_p = \sqrt{\frac{\lambda+2\mu+\alpha^2 Q}{\rho}} \tag{65a}$$

$$c_d = \sqrt{\frac{\lambda+2\mu+\alpha^2 Q}{\rho}}\sqrt{\frac{1}{\dfrac{m}{\rho}\dfrac{\lambda+2\mu+\alpha^2 Q}{Q}+\dfrac{\lambda+2\mu+\alpha^2 Q}{\lambda+2\mu}\dfrac{\alpha^2 m-2\alpha\rho_f}{\rho}+\dfrac{\alpha^2 Q}{\lambda+2\mu}}} \tag{65b}$$

$$\eta_p = \xi_p \approx 0 \tag{65c}$$

$$\eta_d = \xi_d = \frac{1}{2\kappa m}\frac{\lambda+2\mu+\alpha^2 Q}{\lambda+2\mu+\alpha^2 Q-2\alpha Q\dfrac{\rho_f}{m}+\alpha^2 Q\dfrac{\rho}{m}\dfrac{Q}{\lambda+2\mu+\alpha^2 Q}} \tag{65d}$$

which result can also be expressed as:

$$\frac{1}{c_{p,d}^2} = \frac{1}{2}\left(a_6 \mp \frac{a_4}{2a_7}\right) \tag{65e}$$

$$\eta_d = \xi_d = \frac{1}{\kappa}\frac{a_7}{a_6+\dfrac{a_4}{2a_7}} \tag{65f}$$

$$\eta_p = \xi_p = 0 \tag{65g}$$

where a_4, a_6, a_7 are defined in eqns (33) and (35). Similarly, we deduce that:

$$\lambda_3 = \frac{1}{c_s}\left[s + \frac{\eta_s s}{\varepsilon\eta_s + s}\right], \tag{66}$$

where

$$c_s = \frac{1}{1-\frac{1}{2}\frac{\rho_f^2}{\rho m}}\sqrt{\frac{\mu}{\rho}}, \tag{67a}$$

$$\eta_s = \frac{1}{2\kappa m}\frac{\rho_f^2}{m\rho - \frac{1}{2}\rho_f^2}, \tag{67b}$$

$$\varepsilon = \frac{2\rho m}{\rho_f^2} - 1. \tag{67c}$$

Armed with these alternative approximations for $\lambda_1, \lambda_2, \lambda_3$ and $\lambda_1^2 - \lambda_2^2$, which gives a technique for handling difficulty in treating the physical domain Green's function for the general case, and together with familiar Laplace transform formulas, we start to investigate the desired real-time fundamental solution.

Function G_{ij}

Equation (38), which has been deliberately recast for approximation, is an exact expression for $\tilde{G}_{ij}$ in transform domain, where all the constants a, d_1, d_2, d_3, d_4, d_{51}, d_{52}, d_{61}, d_{62}, e_1, e_2, e_{31}, e_{32}, b_1, b_2 are the same as defined in eqns (39).

Substituting from eqn (62) into eqn (38) and bearing in mind eqns (64) and (66), with some algebra leads to:

$$\begin{aligned}\tilde{G}_{ij} = &\left\{f_1\frac{1}{s(s+a)(s+b)} + f_2\frac{1}{s^2(s+a)(s+b)} + f_3\frac{1}{s^3(s+a)(s+b)} + f_4\frac{1}{s^3(s+a)}\right.\\ &\left.+f_5\frac{1}{s+b} + f_6\frac{1}{s(s+b)} + f_7\frac{1}{s^2(s+b)} + f_8\frac{1}{s} + f_9\frac{1}{s^3}\right\}e^{-\lambda_1 r} + \left\{f_{10}\frac{1}{s(s+b)} + f_{11}\frac{1}{s+b}\right.\\ &\left.+f_{12}\frac{1}{s}\right\}\frac{1}{\lambda_1}e^{-\lambda_1 r} + \left\{g_1\frac{1}{s(s+a)(s+b)} + g_2\frac{1}{s^2(s+a)(s+b)} + g_3\frac{1}{s^3(s+a)(s+b)}\right.\\ &\left.+g_4\frac{1}{s^3(s+a)} + g_5\frac{1}{s+b} + g_6\frac{1}{s(s+b)} + g_7\frac{1}{s^2(s+b)} + g_8\frac{1}{s} + g_9\frac{1}{s^2} + g_{10}\frac{1}{s^3}\right\}e^{-\lambda_2 r}\\ &+\left\{h_1\frac{1}{s} + h_2\frac{1}{s^2} + h_3\frac{1}{s^3} + h_4\frac{1}{s(s+d)} + h_5\frac{1}{s^3(s+a)}\right\}e^{-\lambda_3 r},\end{aligned} \tag{68}$$

where

$$d = \frac{1}{\kappa m} \qquad f_1 = -\frac{A_{ij}d_1}{c} \qquad h_1 = D_{ij} - \frac{C_{ij}}{\mu}$$

$$g_1 = \frac{A_{ij}d_1}{c} \qquad f_2 = -\frac{A_{ij}d_2}{c} \qquad h_2 = -\frac{B_{ij}}{\sqrt{\mu\rho}}$$

$$g_2 = \frac{A_{ij}d_2}{c} \qquad f_3 = -\frac{A_{ij}d_3}{c} \qquad h_3 = -A_{ij}b_2$$

$$g_3 = \frac{A_{ij}d_3}{c} \qquad f_4 = -A_{ij}d_{62} \quad h_4 = -\frac{B_{ij}}{2\mu^{1/2}\rho^{3/2}}\frac{\rho_f^2}{m}$$

$$g_4 = A_{ij}d_{61} \qquad f_5 = -\frac{C_{ij}e_2}{c} \quad h_5 = -A_{ij}b_1$$

$$g_5 = \frac{C_{ij}e_2}{c} \qquad f_6 = -\frac{C_{ij}e_1}{c}$$

$$g_6 = \frac{C_{ij}e_1}{c} + \frac{B_{ij}e_2c_p}{c} \qquad f_7 = -\frac{A_{ij}d_4}{c}$$

$$g_7 = \frac{A_{ij}d_4}{c} + \frac{B_{ij}e_1c_p}{c} \qquad f_8 = -C_{ij}e_{31}$$

$$g_8 = C_{ij}e_{32} \qquad f_9 = -A_{ij}d_{52}$$

$$g_9 = B_{ij}e_{32}c_p \qquad f_{10} = -\frac{B_{ij}e_1}{c}$$

$$g_{10} = A_{ij}d_{51} \qquad f_{11} = -\frac{B_{ij}e_2}{c}$$

$$f_{12} = -B_{ij}e_{31}. \tag{69}$$

Now we can invert eqn (68) to arrive at the following transient solution:

$$\begin{aligned}
G_{ij} = {} & \int_{r/c_d}^{t} [P_{11}e^{-b(t-\tau)} + P_{12}]e^{-\eta_d\tau} I_0\left(\xi_d\sqrt{\tau^2 - r^2/c_d^2}\right) d\tau H(t-r/c_d) \\
& + \{P_{21}e^{-a(t-r/c_d)} + P_{22}e^{-b(t-r/c_d)} + P_{23} + P_{24}(t-r/c_d) + P_{25}(t-r/c_d)^2\} \\
& \times e^{-\eta_d r/c_d} H(t-r/c_d) + \int_{r/c_d}^{t} \{P_{21}e^{-a(t-\tau)} + P_{22}e^{-b(t-\tau)} + P_{23} \\
& + P_{24}(t-\tau) + P_{25}(t-\tau)^2\} e^{-\eta_d\tau} \frac{\xi_d r/c_d}{\sqrt{\tau^2 - r^2/c_d^2}} I_1\left(\xi_d\sqrt{\tau^2 - r^2/c_d^2}\right) d\tau H(t-r/c_d) \\
& + \{P_{31}e^{-a(t-r/c_p)} + P_{32}e^{-b(t-r/c_p)} + P_{33} + P_{34}(t-r/c_p) + P_{35}(t-r/c_p)^2\} \\
& \times H(t-r/c_p) + \int_{r/c_s}^{t} \{P_{41}e^{-a(t-\tau)} + P_{42}e^{-d(t-\tau)} + P_{43} + P_{44}(t-\tau) + P_{45}(t-\tau)^2\} \\
& \times e^{-\varepsilon\eta_s(\tau - r/c_s)} e^{-\eta_s r/c_s} \sqrt{\varepsilon}\eta_s \sqrt{\frac{r}{(\tau - r/c_s)c_s}} I_1\left(2\sqrt{\varepsilon}\eta_s\sqrt{\frac{r(\tau - r/c_s)}{c_s}}\right) d\tau H(t-r/c_s) \\
& + \{P_{41}e^{-a(t-r/c_s)} + P_{42}e^{-d(t-r/c_s)} + P_{43} + P_{44}(t-r/c_s) \\
& + P_{45}(t-r/c_s)^2\} e^{-\eta_s r/c_s} H(t-r/c_s)
\end{aligned} \tag{70}$$

where:

$$P_{11} = \left(-\frac{f_{10}}{b} + f_{11}\right)c_d$$

$$P_{12} = \left(\frac{f_{10}}{b} + f_{12}\right)c_d$$

$$P_{21}=\frac{f_1}{a(a-b)}-\frac{f_2}{a^2(a-b)}+\frac{f_3}{a^3(a-b)}-\frac{f_4}{a^3}$$

$$P_{22}=-\frac{f_1}{b(a-b)}+\frac{f_2}{b^2(a-b)}-\frac{f_3}{b^3(a-b)}+f_5-\frac{f_6}{b}+\frac{f_7}{b^2}$$

$$P_{23}=\frac{f_1}{ab}-\frac{f_2(a+b)}{a^2b^2}+\frac{f_3(a^3-b^3)}{a^3b^3(a-b)}+\frac{f_4}{a^3}+\frac{f_6}{b}-\frac{f_7}{b^2}+f_8$$

$$P_{24}=\frac{f_2}{ab}-\frac{f_3(a+b)}{a^2b^2}-\frac{f_4}{a^2}+\frac{f_7}{b}$$

$$P_{25}=\frac{f_3}{2ab}+\frac{f_4}{2a}+\frac{f_9}{2}$$

$$P_{31}=\frac{g_1}{a(a-b)}-\frac{g_2}{a^2(a-b)}+\frac{g_3}{a^3(a-b)}-\frac{g_4}{a^3}$$

$$P_{32}=-\frac{g_1}{b(a-b)}+\frac{g_2}{b^2(a-b)}-\frac{g_3}{b^3(a-b)}+g_5-\frac{g_6}{b}+\frac{g_7}{b^2}$$

$$P_{33}=\frac{g_1}{ab}-\frac{g_2(a+b)}{a^2b^2}+\frac{g_3(a^3-b^3)}{a^3b^3(a-b)}+\frac{g_4}{a^3}+\frac{g_6}{b}-\frac{g_7}{b^2}+g_8$$

$$P_{34}=\frac{g_2}{ab}-\frac{g_3(a+b)}{a^2b^2}-\frac{g_4}{a^2}+\frac{g_7}{b}+g_9$$

$$P_{35}=\frac{g_3}{2ab}+\frac{g_4}{2a}+\frac{g_{10}}{2}, \tag{71a}$$

$$P_{41}=-\frac{h_5}{a^3}$$

$$P_{42}=-\frac{h_4}{d}$$

$$P_{43}=\frac{h_5}{a^3}+h_1+\frac{h_4}{d}$$

$$P_{44}=-\frac{h_5}{a^2}+h_2$$

$$P_{45}=\frac{h_5}{2a}+\frac{h_3}{2}. \tag{71b}$$

Looking now at eqn (70), we observe that the Green's tensor G_{ij} splits up into three waves: diffusive, pressure wave and transverse wave with speeds c_d, c_p, c_s and viscous dissipation factors η_d, η_p, η_s, respectively; obviously, no pulse terms exist in eqn (70).

Function G_{4i}

The exact expression for $\tilde{G}_{4i}$ can be found in eqn (46). We obtain—on introducing $\lambda_1^2-\lambda_2^2 \approx cs(s+b)$ into this relation and keeping in mind $\lambda_1 \approx (1/c_d)\sqrt{(s+\eta_d)^2-\xi_d^2}$, $\lambda_2 \approx (s/c_p)$—the following approximate expressions for $\tilde{G}_{4i}$:

$$\tilde{G}_{4i}=\left(f_1\frac{1}{s+b}+f_2\frac{1}{s(s+b)}\right)e^{-\lambda_1 r}+\left(f_3 s+f_4+f_5\frac{1}{s+b}\right)\frac{1}{\lambda_1}e^{-\lambda_1 r}$$
$$+\left(g_1+g_2\frac{1}{s}+g_3\frac{1}{s+b}+g_4\frac{1}{s(s+b)}\right)e^{(-r/c_p)s}, \tag{72}$$

where

$$f_1 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^3}\frac{d_2}{c}$$

$$f_2 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^3}\frac{d_1}{c}$$

$$f_3 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(e_{11}+\frac{e_5}{c}\right)$$

$$f_4 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(e_{21}+\frac{e_4-e_5b}{c}\right)$$

$$f_5 = -\frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\frac{e_3-e_4b+e_5b^2}{c}$$

$$g_1 = \frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\left(e_{12}+\frac{e_5}{c}\right)c_p$$

$$g_2 = \frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}e_{22}c_p$$

$$g_3 = \frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\frac{1}{c}\left(e_4c_p-e_5c_pb+\frac{d_2}{r}\right)$$

$$g_4 = \frac{1}{4\pi(\lambda+2\mu)}\frac{x_i}{r^2}\frac{1}{c}\left(e_3c_p+\frac{d_1}{r}\right). \tag{73}$$

It is now possible to perform the inverse Laplace transformation. Thereafter, making use of the theorem of convolution and taking into consideration other Laplace transform properties, and, after a Laplace transformation is taken, we have the following solution:

$$\begin{aligned}G_{4i} = {} & (P_{11}e^{-b(t-r/c_d)}+P_{12})e^{-\eta_d r/c_d}H(t-r/c_d)+\int_{r/c_d}^{t}(P_{11}e^{-b(t-\tau)}+P_{12}) \\ & \times e^{-\eta_d\tau}\frac{\xi_d r/c_d}{\sqrt{\tau^2-r^2/c_d^2}}I_1\left(\xi_d\sqrt{\tau^2-r^2/c_d^2}\right)d\tau H(t-r/c_d)+\int_{r/c_d}^{t}P_{21}e^{-b(t-\tau)}e^{-\eta_d\tau} \\ & \times I_0\left(\xi_d\sqrt{\tau^2-r^2/c_d^2}\right)d\tau H(t-r/c_d)+P_{22}e^{-\eta_d r/c_d}\delta(t-r/c_d)+P_{23}e^{-\eta_d t} \\ & \times I_0\left(\xi_d\sqrt{t^2-r^2/c_d^2}\right)H(t-r/c_d)+P_{24}e^{-\eta_d t}I_1\left(\xi_d\sqrt{t^2-r^2/c_d^2}\right)\frac{t}{\sqrt{t^2-r^2/c_d^2}} \\ & \times H(t-r/c_d)+P_{31}e^{-b(t-r/c_p)}H(t-r/c_p)+P_{32}H(t-r/c_p)+P_{33}\delta(t-r/c_p),\end{aligned} \tag{74}$$

where:

$$P_{11} = f_1-\frac{f_2}{b} \quad P_{21} = f_5c_d \quad P_{31} = g_3-\frac{g_4}{b}$$

$$P_{12} = \frac{f_2}{b} \quad P_{22} = f_3c_d \quad P_{32} = g_2+\frac{g_4}{b}$$

$$P_{23} = (-f_3\eta_d+f_4)c_d \quad P_{33} = g_1.$$

$$P_{24} = f_3c_d\xi_d. \tag{75}$$

The solution consists of two waves: the first compressive wave and the second compressive wave. Two pulse terms appear due to the arrival of P_1 wave and P_2 wave.

Function G_{i4}

Since:

$$\tilde{G}_{i4} = \frac{1}{s}\tilde{G}_{4i}.$$

From eqn (72) we directly write $\tilde{G}_{i4}$ as:

$$\tilde{G}_{i4} = \left(f_1\frac{1}{s(s+b)}+f_2\frac{1}{s^2(s+b)}\right)e^{-\lambda_1 r}+\left(f_3+f_4\frac{1}{s}+f_5\frac{1}{s(s+b)}\right)\frac{1}{\lambda_1}e^{-\lambda_1 r}$$
$$+\left(g_1\frac{1}{s}+g_2\frac{1}{s^2}+g_3\frac{1}{s(s+b)}+g_4\frac{1}{s^2(s+b)}\right)e^{(-r/c_p)s} \quad (76)$$

where $f_1, f_2, f_3, f_4, f_5, g_1, g_2, g_3, g_4$ take the same values as eqn (73). After an inverse Laplace transform performed on eqn (76) we find that:

$$G_{i4} = [P_{11}e^{-b(t-r/c_d)}+P_{12}+P_{13}(t-r/c_d)]e^{-\eta_d r/c_d}H(t-r/c_d)+\int_{r/c_d}^{t}[P_{11}e^{-b(t-\tau)}$$
$$+P_{12}+P_{13}(t-\tau)]e^{-\eta_d\tau}\frac{\xi_d r/c_d}{\sqrt{\tau^2-r^2/c_d^2}}I_1\left(\xi_d\sqrt{\tau^2-r^2/c_d^2}\right)d\tau H(t-r/c_d)$$
$$+\int_{r/c_d}^{t}[P_{21}e^{-b(t-\tau)}+P_{22}]e^{-\eta_d\tau}I_0\left(\xi_d\sqrt{\tau^2-r^2/c_d^2}\right)d\tau H(t-r/c_d)$$
$$+P_{23}e^{-\eta_d t}I_0\left(\xi_d\sqrt{t^2-r^2/c_d^2}\right)H(t-r/c_d)+(P_{31}e^{-b(t-r/c_p)}+P_{32}$$
$$+P_{33}(t-r/c_p))H(t-r/c_p), \quad (77)$$

where

$$P_{11} = -\frac{f_1}{b}+\frac{f_2}{b^2} \qquad P_{31} = -\frac{g_3}{b}+\frac{g_4}{b^2}$$
$$P_{12} = \frac{f_1}{b}-\frac{f_2}{b^2} \qquad P_{32} = g_1+\frac{g_3}{b}-\frac{g_4}{b^2}$$
$$P_{13} = \frac{f_2}{b} \qquad P_{33} = g_2+\frac{g_4}{b}$$
$$P_{21} = -\frac{f_5 c_d}{b} \qquad P_{23} = f_3 c_d$$
$$P_{22} = \left(f_4+\frac{f_5}{b}\right)c_d. \quad (78)$$

This once again predicts the existence of two dilatational waves.

Function G_{44}

Now we shall apply the Laplace inversion on eqn (56). To that end, we put in eqn (56) $\lambda_1^2-\lambda_2^2 \approx cs(s+b)$, and bearing in mind $\lambda_1 \approx (1/c_d)\sqrt{(s+\eta_d)^2-\xi_d^2}$, $\lambda_2 \approx s/c_p$, we arrive at:

$$\tilde{G}_{44} = \left(f_1+f_2\frac{1}{s}+f_3\frac{1}{s+b}+{}_4\frac{1}{s(s+b)}\right)e^{-\lambda_1 r}+\left(g_1+g_2\frac{1}{s}+g_3\frac{1}{s+b}+g_4\frac{1}{s(s+b)}\right)e^{-\lambda_2 r}$$
$$(79)$$

where:

$$f_1 = \frac{m}{8\pi r} + \frac{d_3}{8\pi rc} \quad g_1 = \frac{m}{8\pi r} - \frac{d_3}{8\pi rc}$$
$$f_2 = \frac{1}{8\pi r\kappa} \quad g_2 = \frac{1}{8\pi r\kappa}$$
$$f_3 = \frac{d_2 - d_3 b}{8\pi rc} \quad g_3 = \frac{-d_2 + d_3 b}{8\pi rc}$$
$$f_4 = \frac{d_1}{8\pi rc} \quad g_4 = -\frac{d_1}{8\pi rc}. \tag{80}$$

Inverting this result by means of the Laplace transform for the above well-posed approximate equation, we obtain the solution:

$$\begin{aligned} G_{44} = {} & P_{11} e^{-\eta_d t}\delta(t - r/c_d) + P_{12} e^{-\eta_d t}\frac{1}{\sqrt{t^2 - r^2/c_d^2}} I_1(\xi_d\sqrt{t^2 - r^2/c_d^2})H(t - r/c_d) \\ & + \int_{r/c_d}^{t} (P_{13} e^{-b(t-\tau)} + P_{14}) e^{-\eta_d \tau} \frac{\xi_d r/c_d}{\sqrt{\tau^2 - r^2/c_d^2}} I_1(\xi_d\sqrt{\tau^2 - r^2/c_d^2}) \\ & \times d\tau H(t - r/c_d) + (P_{13} e^{-b(t - r/c_d)} + P_{14}) e^{-\eta_d r/c_d} H(t - r/c_d) \\ & + P_{21}\delta(t - r/c_p) + (P_{22} e^{-b(t - r/c_p)} + P_{23}) H(t - r/c_p), \end{aligned} \tag{81}$$

where:

$$P_{11} = f_1 \quad P_{21} = g_1$$
$$P_{12} = f_1 \xi_d \frac{r}{c_d} \quad P_{22} = g_3 - \frac{g_4}{b}$$
$$P_{13} = f_3 - \frac{f_4}{b} \quad P_{23} = g_2 + \frac{g_4}{b}$$
$$P_{14} = f_2 + \frac{f_4}{b}. \tag{82}$$

Obviously, the shear wave disappeared in the pressure field due to fluid injection.

NUMERICAL RESULTS

We now wish to examine the accuracy of the analytical transient Green's functions for both the limiting case and the general case. Naturally, the simplest way is to compare the analytical results with those obtained by an accurate numerical inversion of the Laplace transform solutions presented in eqn (15). All plots are presented nondimensionally.

Numerical results for the limiting case

Following the definition of nondimensional parameters of eqns (11), the material parameters for Berea sandstone (Yew and Jogi, 1978; Burridge and Vargas, 1979) will be recast in nondimensional form as follows: $\lambda^* = 0.1715$, $\mu^* = 0.3007$, $Q^* = 0.3742$, $\rho^* = 1.0$, $\rho_f^* = 0.4325$, $m^* = 2.3006$, $\kappa^* = 1.0$, $\alpha = 0.779$.

Figures (1)–(4) depict the three-dimensional fundamental solution components G_{ij}, G_{i4}, G_{4j}, G_{44}, where $i, j = 1, 2, 3$ for the limiting case (early time solution). We assume that the applied force point (or fluid source point) is located at $(0, 0, 0)$, the receiver is chosen at nondimensional coordinate (0.1, 0.15, 0.2). The nondimensional velocities of the three waves are approximately $c_p = 1.0$ (pressure wave or P_1 wave); $c_d = 0.368$ (diffusive wave or P_2 wave) and $c_s = 0.572$ (shear wave). Thus, they arrive at the receiver (0.1, 0.15, 0.2) at $t_p = 0.268$ (pressure wave), $t_s = 0.471$ (shear wave), and $t_d = 0.732$ (diffusive wave). All three arrival times can be detected on Figs (1) by sudden changes appearing in the displacement due to the point force. Only two arrival times, corresponding to the first compressional wave and second compressional wave, can be identified on Fig. (2) by sudden changes in displacement in the x_2-direction due to point source injection, and by two pulses appearing in the excessive pore fluid pressure on Figs (3) and (4) due to point force in the x_2-direction and point source injection, respectively. These pulses take the form of Dirac delta function $\delta(t-r/c)$. It is of some interest to examine the values of dissipation factors of the three waves. They are $\eta_d = 0.2336$ (diffusive wave), $\eta_p = 0.00294$ (pressure wave), $\eta_s = 0.0192$ (shear wave). This confirms Biot's finding that the waves of the second kind (diffusive wave with large value of η_d) are highly attenuated and the waves of the first kind are true waves (pressure wave with negligible small value of η_p).

In order to examine the accuracy of the analytical solutions, all the figures are plotted as a comparison with the results of the numerical inversion of the Laplace transform solutions. The comparison demonstrates that the analytical solutions can capture the salient nature and characteristics of waves in porous media near the arrival time. However, as predicted earlier, the accuracy of the analytical solutions deteriorates and deviates as time increases. Fortunately the drawbacks of this early time solution were overcome by a companion solution for the general case, which is to be studied graphically next.

Numerical results for the general case

The material parameters for Pecos sandstone (Yew and Jogi, 1978; Burridge and Vargas, 1979) are recast in nondimensional form as follows: $\lambda^* = 0.1286$, $\mu^* = 0.2746$, $Q^* = 0.4679$, $\rho^* = 1.$, $\rho_f^* = 0.4399$, $m^* = 2.256$, $\kappa^* = 1.$, $\alpha = 0.83$.

Figures (5)–(8) depict the three-dimensional fundamental solution components for the general case. The applied force point (or fluid source point) is located at (0, 0, 0) and the receiver is chosen at nondimensional coordinate (1, 2, 3). The nondimensional wave velocities are approximately $c_p = 1.0$ (pressure wave or P_1 wave); $c_d = 0.3918$ (diffusive wave or P_2 wave); $c_s = 0.5475$ (shear wave). In view of the above, the nondimensional time required for the three waves to reach the receivers are $t_p = 3.7417$ (pressure wave), $t_s = 6.8344$ (shear wave), and $t_d = 9.5488$ (diffusive wave). In all the figures, excellent agreement of analytical solution and numerical Laplace inversion is seen.

Figures (6)–(8) clearly demonstrate the existence of two wave fronts (P_1 wave and P_2 wave); these fronts propagate with speeds c_p and c_d, respectively. An interesting feature is the presence of two pulses in those Green's functions which define the pressure due to point force or point source injection, as illustrated by Figs (7)–(8). These pulses, in the form of the Dirac delta function for the three-dimensional case, are associated with the arrival of the two dilatational waves.

Figures (5a) and (5b) indicate that the displacement component of Green's function due to point force contains three wave fronts, two dilatational waves and one rotational wave. They propagate with speeds c_p (pressure wave), c_d (diffusive wave) and c_s (shear wave), respectively. Analytical solutions for the general case have shown that the propagation of the fast wave (P_1) is characterized by nearly compatible deformations of the solid and fluid phases; very little viscous attenuation is induced with practically negligibly small dissipation factor $\eta_p \approx 0$. This strongly confirms Biot's (1956a,b) finding that the waves of the first kind are true waves, the dispersion is practically negligible. Therefore the P_1 wave can be detected in the far field as well as the near field of the source. The propagation of the slow wave (P_2) is characterized by the fluid and solid dilatations being nearly 180 degrees out of phase and the propagation is strongly attenuated with a high dissipative factor $\eta_d = 0.242$, and hence it can be detected only at a close proximity of the source. As the disturbance moves into the media, the second wave front slows down and eventually disappears. The dissipative factor $\eta_s = 0.0199$ for the shear wave is much smaller than for the P_2 wave. Obviously the

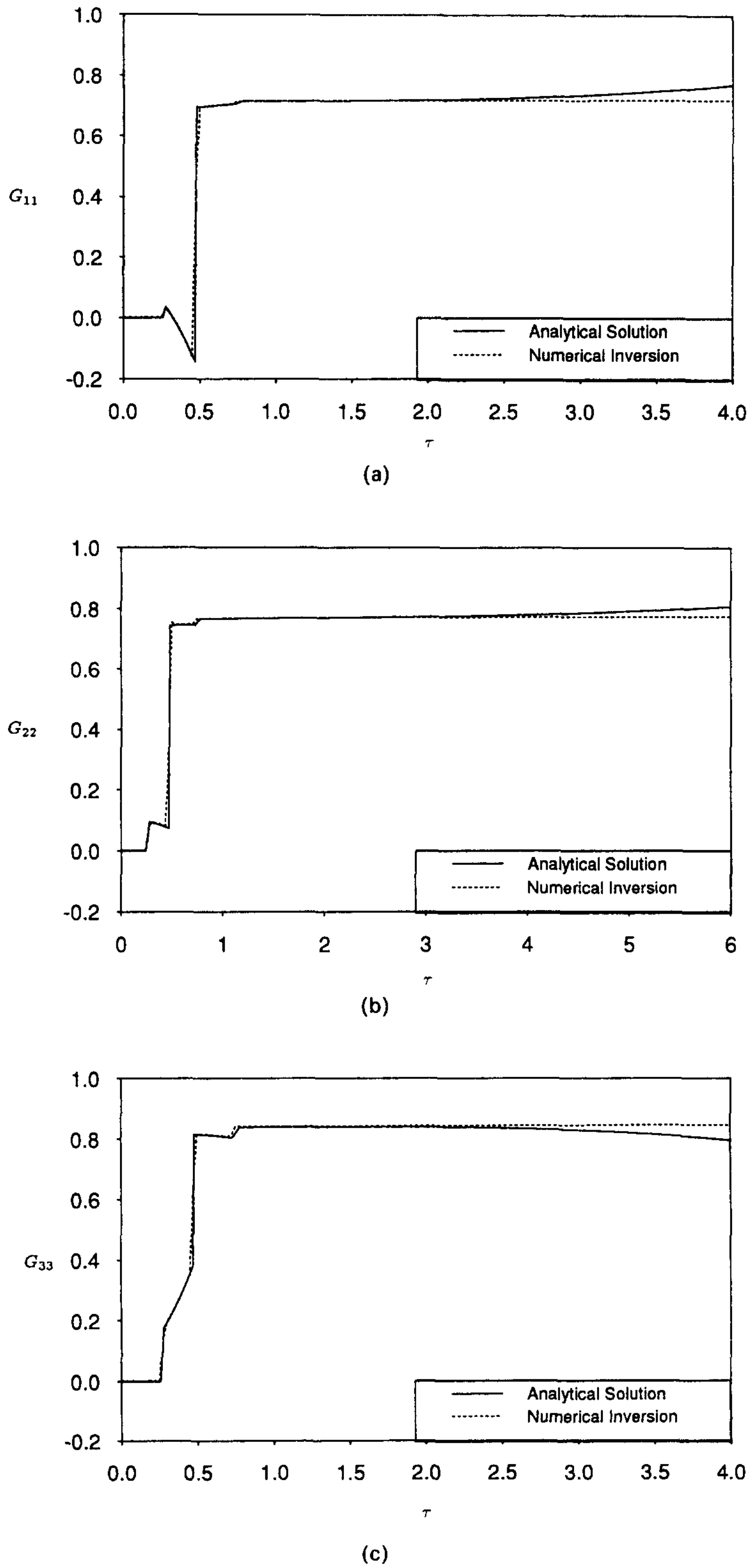
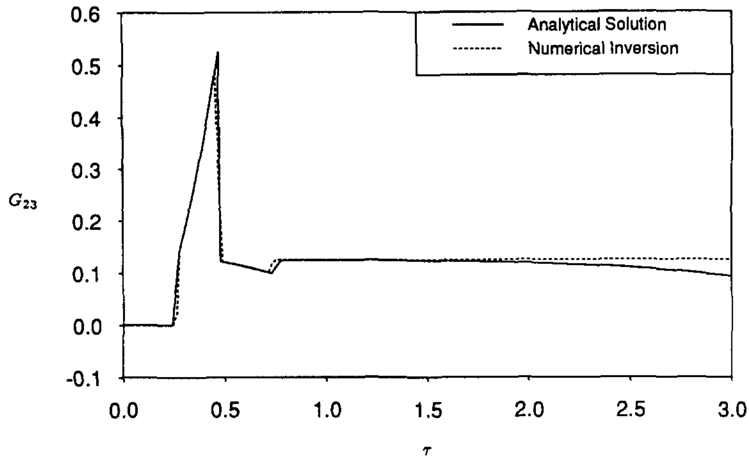


Fig. 1. Three-dimensional displacement time history at $\vec{\xi} = (0.1, 0.15, 0.2)$ due to point force at $(0, 0, 0)$.



(d)

Fig. 1—*continued.*

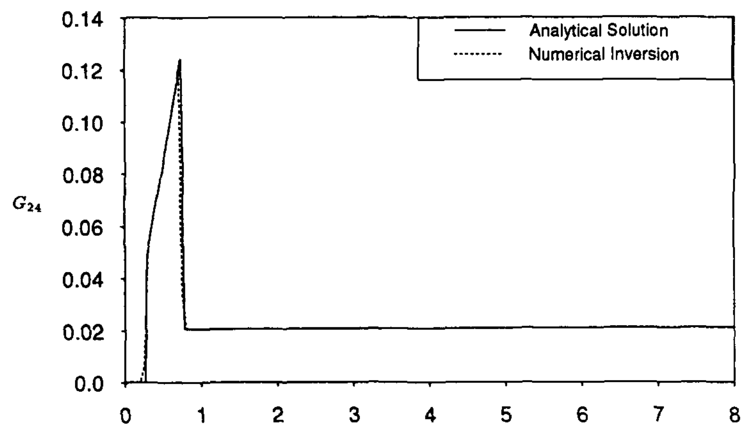


Fig. 2. Three-dimensional displacement time history at $\vec{\xi} = (0.1, 0.15, 0.2)$ due to fluid injection at $(0, 0, 0)$.

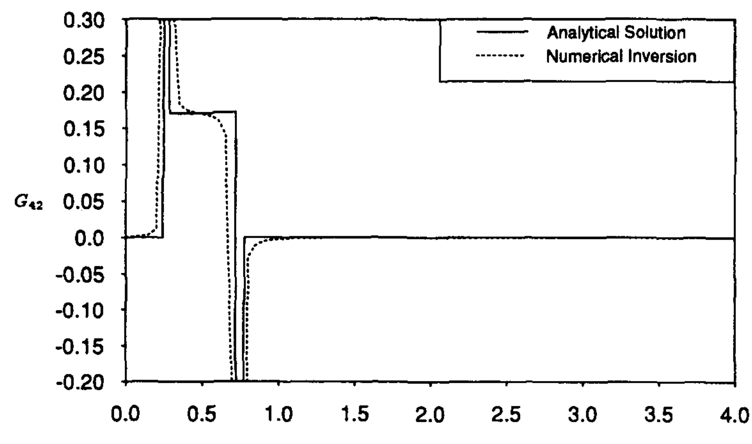


Fig. 3. Three-dimensional pressure time history at $\vec{\xi} = (0.1, 0.15, 0.2)$ due to point force at $(0, 0, 0)$.

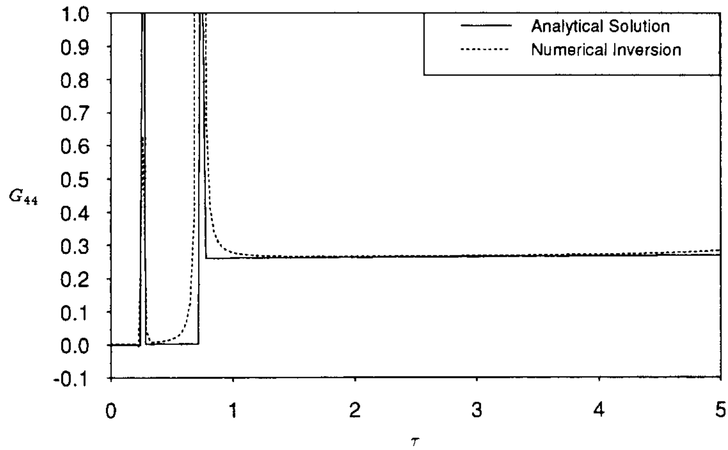


Fig. 4. Three-dimensional pressure time history at $\vec{\xi} = (0.1, 0.15, 0.2)$ due to fluid injection at $(0, 0, 0)$.

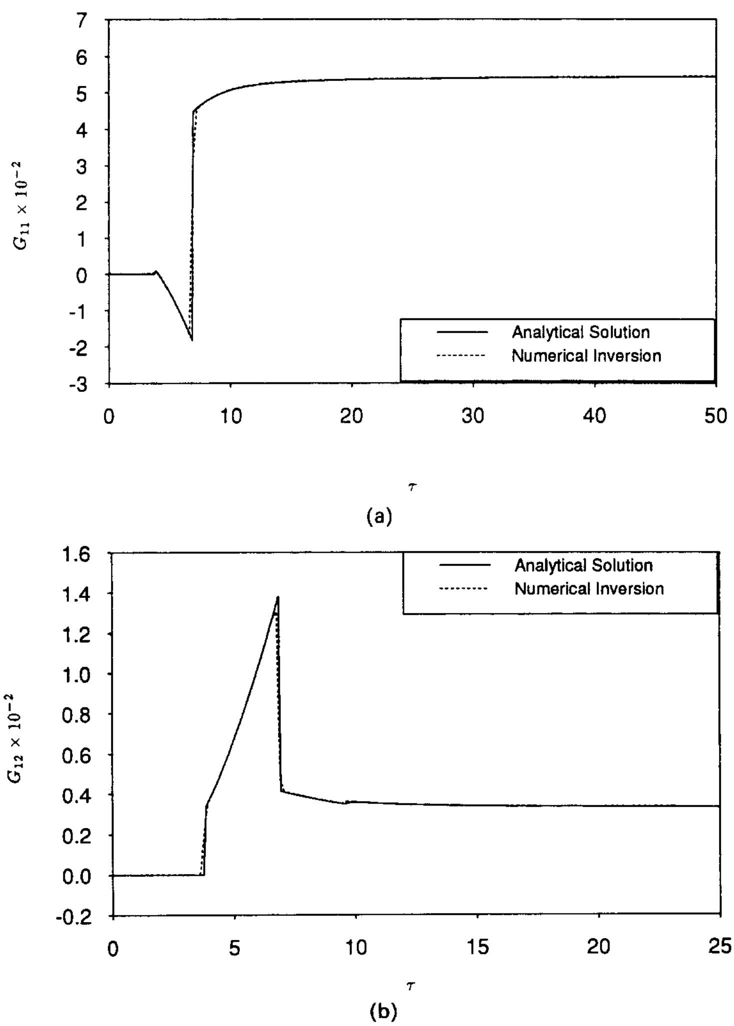


Fig. 5. Three-dimensional displacement time history at $\vec{\xi} = (1, 2, 3)$ due to point force at $(0, 0, 0)$.

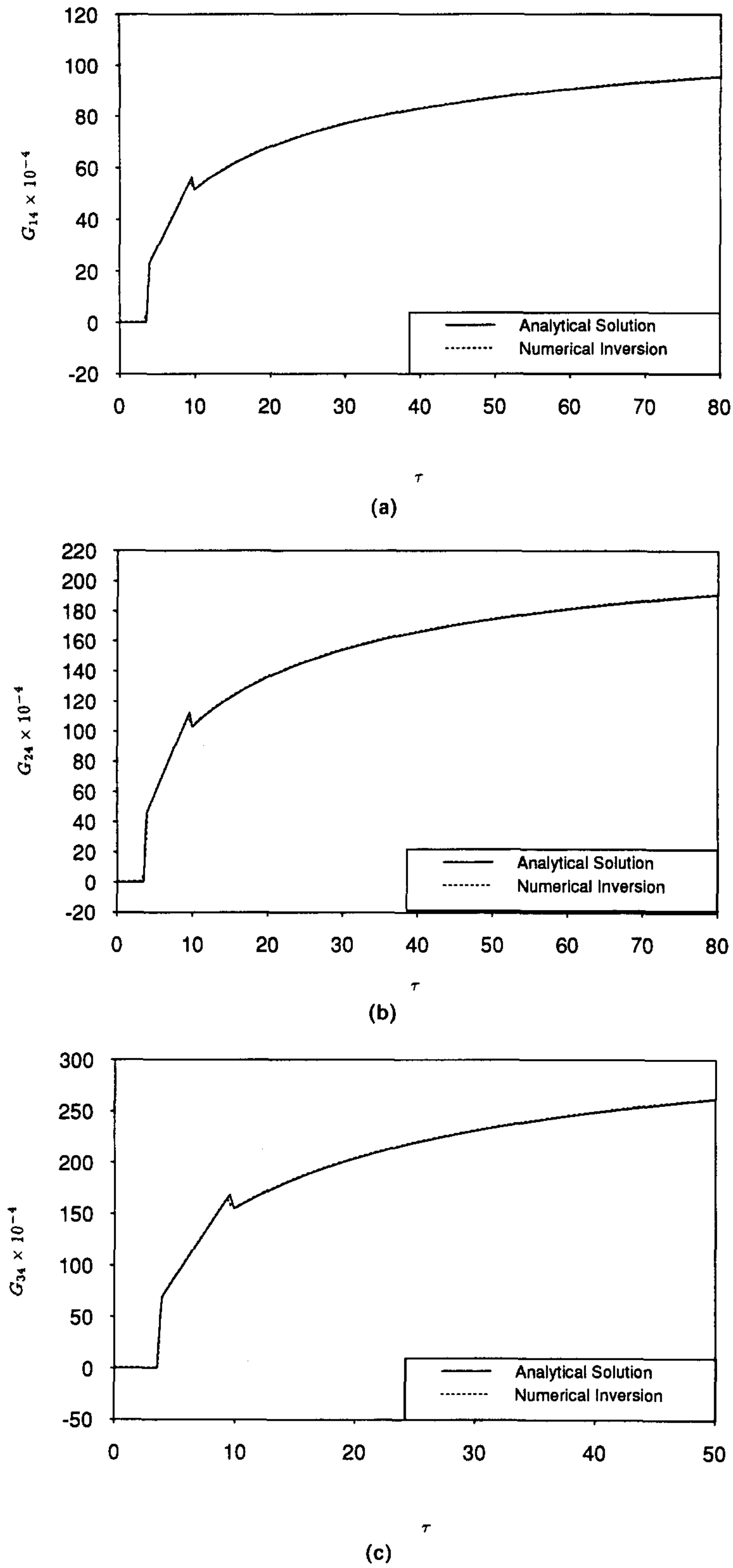


Fig. 6. Three-dimensional displacement time history at $\vec{\xi} = (1, 2, 3)$ due to fluid injection at $(0, 0, 0)$.

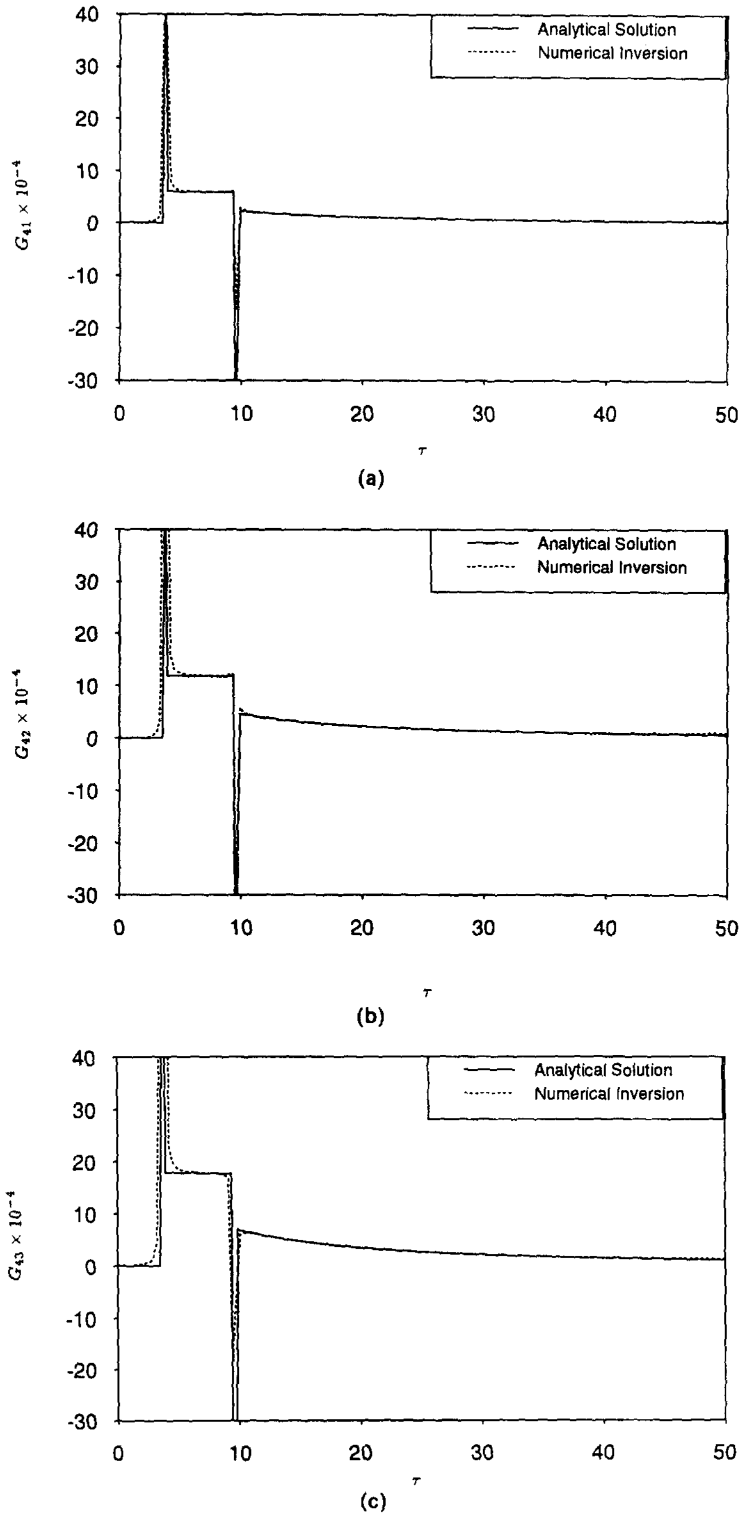


Fig. 7. Three-dimensional pressure time history at $\xi = (1, 2, 3)$ due to point force at $(0, 0, 0)$.

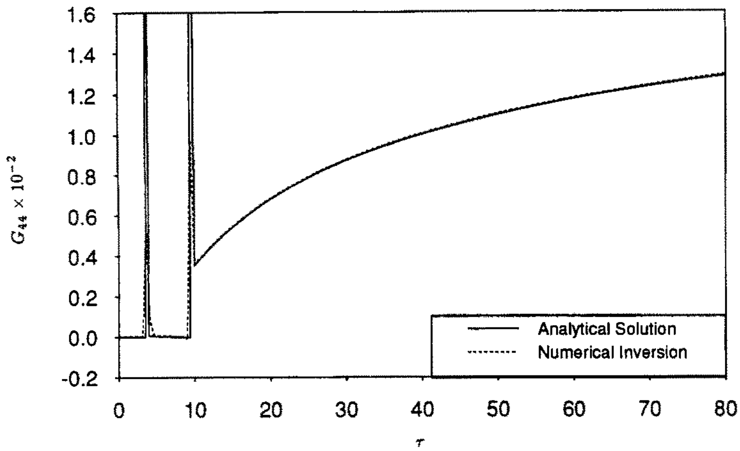


Fig. 8. Three-dimensional pressure time history at $\bar{\xi} = (1, 2, 3)$ due to fluid injection at $(0, 0, 0)$.

attenuation enters through the inclusion of a damping term in the original equations due to the difference of solid and fluid velocities.

We turn next to the discussion of the interaction between the wave propagation and diffusion process. The fact that no diffusion takes place before the arrival of P_2 wave and it starts right after the arrival of P_2 wave is evident from Figs (7) and (8). This is primarily due to high viscous attenuation of P_2 wave which is in the nature of a diffusion process, and the propagation is closely analogous to heat conduction or related to consolidation.

An inspection of the two-dimensional Green's function plots in Chen (1993a) and the three-dimensional plots in this work reveals that there is a significant difference between the two cases, i.e. the response in three dimensions exhibits no tail, which do exist in the two-dimensional case, when plotted with respect to time. The reason for this phenomena is that in the three-dimensional case, the impulse at the origin reaches a specified receiver in the domain and is gone, thus immediately thereafter the response starts to decrease. In the two-dimensional case, the disturbance keeps reaching the receiver and the response continues to exist after the arrival or the first disturbance.

CONCLUSION

A complete set of closed form transient fundamental solutions are constructed for Biot's three-dimensional full dynamic poroelasticity. Verification of the solutions is performed by comparing with Laplace transform domain solutions. Excellent agreement is found for the solutions to the general case, while solutions to the limiting case can capture the salient characteristics of the waves at early time. This leads to the following conclusions: the transient Green's functions presented in this paper and a companion paper (Chen, 1993a) may be used effectively as kernal functions of the time domain boundary element method.

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